

Game-Theoretic Pricing for Video Streaming in Mobile Networks

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Abstract—Mobile phones are among the most popular consumer devices, and the recent developments of 3G networks and smart phones enable users to watch video programs by subscribing data plans from service providers. Due to the ubiquity of mobile phones and phone-to-phone communication technologies, data-plan subscribers can redistribute the video content to nonsubscribers. Such a redistribution mechanism is a potential competitor for the mobile service provider and is very difficult to trace given users' high mobility. The service provider has to set a reasonable price for the data plan to prevent such unauthorized redistribution behavior to protect or maximize his/her own profit. In this paper, we analyze the optimal price setting for the service provider by investigating the equilibrium between the subscribers and the secondary buyers in the content-redistribution network. We model the behavior between the subscribers and the secondary buyers as a noncooperative game and find the optimal price and quantity for both groups of users. Based on the behavior of users in the redistribution network, we investigate the evolutionarily stable ratio of mobile users who decide to subscribe to the data plan. Such an analysis can help the service provider preserve his/her profit under the threat of the redistribution networks and can improve the quality of service for end users.

Index Terms—Game theory, mobile video streaming, pricing.

I. INTRODUCTION

THE explosive advance of multimedia processing technologies are creating dramatic shifts in ways that video content is delivered to and consumed by end users. Also, the increased popularity of wireless networks and mobile devices is drawing lots of attentions on ubiquitous multimedia access in the multimedia community in the past decade. Network service providers and researchers are focusing on developing efficient solutions to ubiquitous access of multimedia data, particularly videos, from everywhere using mobile devices (laptops, personal digital assistants, or smart phones that can access 3G networks) [1], [2]. Mobile-phone users can watch video programs on their devices by subscribing to the data plans from network service providers [3], [4], and they can easily use their programmable hand devices to retrieve and reproduce the video content. To accommodate heterogeneous network conditions and devices, scalable video coding is also widely used in mobile video streaming [5]–[7]. Video applications over mobile devices have drawn lots of attentions in

the research community, such as quality measure [8], [9] and error control [10]. There is also a rich body of literature on user interactions in electronic commerce in wireless networks such as cooperative content caching in wireless ad hoc network [11]–[13] and secure transactions [14]. Therefore, it is important to understand end users' possible actions in order to provide better ubiquitous video access services.

According to a survey on the popularity of mobile devices [15], almost every person has at least one cell phone in developed countries, and video consumption over mobile devices is an emerging trend [16]. With such a high popularity and the convenient phone-to-phone communication technologies, it is very possible for data-plan subscriber to redistribute the video content without authorization. For example, some users who do not subscribe to the data plan may wish to watch television programs while waiting for public transportation, and some of them might want to check news from time to time. Hence, these users have incentives to buy the desired video content from neighboring data subscribers if the cost is lower than the subscription fee charged by the service provider. Unlike generic data, multimedia data can be easily retrieved and modified, which facilitates the redistribution of video content. In addition, subscribers also have incentives to redistribute the content with a price higher than their transmission cost, as long as such an action will not be detected by the content owner. Due to the high-mobility, time-sensitiveness, and small-transmission-range characteristics of mobile devices, each redistribution action only exists for a short period of time and is very difficult to track. Consequently, a better way to prevent copyright infringement is to set a pricing strategy such that no subscriber will have the incentive to redistribute the video.

Nevertheless, the mobile network service provider might be more interested in setting the content price to maximize his/her own profit than protecting copyrights. The service provider's profit can be represented as the total number of subscriptions times the content price. If the content price is high, mobile users have less incentive to subscribe to the data plan, which might result in less subscription. However, on the other hand, the content price in the redistribution network may get higher due to less subscribers and more secondary buyers. In such a case, although a subscriber pays more for the video stream, he/she also gets more compensation by redistributing the data. Hence, setting the content price higher does not necessarily reduce the number of subscriptions, and it is not trivial to find the optimal price that maximizes the service provider's utility.

The service provider, the data-plan subscribers, and the secondary buyers who are interested in the video data interact with each other and influence each other's decisions and performance. In such a scenario, the game theory is a mathematical tool to model and analyzes the strategic interactions among

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rational decision makers. Recently, the game theory has drawn great attention in multimedia signal processing [17], [18]. We first model the user dynamics in the redistribution network as a multiplayer noncooperative game and obtain the equilibrium price from which all users have no incentives to deviate. Hence, such an equilibrium price will serve as the upper bound for the price set by the network service provider to prevent copyright infringement. Due to the small coverage area and the limited power of each mobile device, a subscriber can only sell the content to secondary buyers within his/her transmission range, and the distance between users and the channel conditions dominate users' decisions. Then, we add the service provider as a player to the game to analyze the optimal pricing for the service provider in the video streaming marketing network. Since the mobile users can change their decisions on subscribing or resubscribing, the content owner is interested in the number of subscribers that is stable over the time. Therefore, a robust equilibrium solution is desired for the service provider. Hence, we formulate the video streaming marketing phenomenon as an evolutionary game and derive the evolutionarily stable strategy (ESS) [19] for the mobile users, which is the desired stable equilibrium for the service provider.

The rest of this paper is organized as follows: We introduce the system model in Section II. We then analyze the optimal strategies for all users in the redistribution network and prove the existence of the equilibrium when there is only one secondary buyer in Section III. We then analyze the mixed-strategy equilibrium for the scenario with multiple secondary buyers in Section IV. In Section V, the content owner is also considered as a player who sets the price to maximize his/her payoff but does not prevent the video redistribution among users. Conclusions are drawn in Section VI.

II. SYSTEM MODEL

In this section, we will introduce the channel, transmission, and video rate-distortion models for the transmission of video streams over wireless networks.

The system diagram is shown in Fig. 1. There are N_s subscribers in the network, who are trying to sell the video content to N_b secondary buyers. Here, we assume that the content is redistributed through direct links between the subscribers and the secondary buyers, i.e., these mobile users form an ad hoc network. Given the current technology, such direct link can be Bluetooth or Wi-Fi. At the beginning, each subscriber sends his/her own price per unit transmission power, as well as the probing signal to secondary buyers. Since the price information contains only a few bits, we assume that it can be immediately and perfectly received. The probing signal enables secondary buyers to estimate the maximal achievable transmission rate. A secondary buyer has to decide how much power he/she wants to buy from each subscriber. Since scalable video coding is widely used in mobile video streaming [5], secondary buyers can purchase different coding layers of the video from different subscribers and combine these streams during the decoding process. Any higher layer mechanisms for such wireless ad hoc networks such as bootstrapping algorithms [20], [21] can be applied to the redistribution network and will not change the analysis in the following sections.

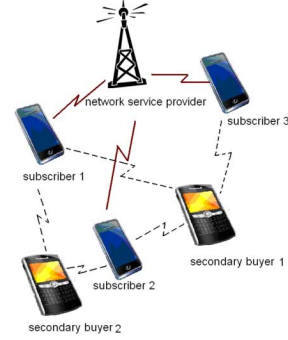


Fig. 1. Example of a mobile video-stream redistribution network.

Assume that the j th secondary buyer purchases a part of the video stream from subscriber S_i with transmission power $P_i^{(j)}$. We assume that there is a channel dedicated for transmissions among users [22], [23] and this channel is a slow-fading channel with channel gain H_{ij} ; the distance between them is d_{ij} , and the variance of the additive white Gaussian noise at the receiver's side is σ^2 [24]. Let N' be the set of subscribers from whom the secondary buyers purchase the video. Assume that the total bandwidth available for the video redistribution network is W , which will be evenly allocated to all N' subscribers from whom secondary buyers purchase the video stream. The signal-to-noise ratio (SNR) and the maximal achievable bit rate of the video stream between S_i and B_j are

$$\begin{aligned} \text{SNR}_{ij} &= \frac{P_i^{(j)} H_{ij}}{\sqrt{d_{ij}} \sigma^2} \\ R_{ij} &= \frac{W}{N' + 1} \log_2 \left(1 + \frac{\text{SNR}_{ij}}{\gamma} \right) \end{aligned} \quad (1)$$

where γ is the capacity gap [25], [26].

For video streaming services, two commonly used objective quality measurements are the video's peak SNR (PSNR) and the streaming delay. Here, we adopt the polynomial delay model as in [27]. The overall delay D_B at the secondary buyers' end is the network delay between the subscribers and the service provider plus the maximal processing time of the subscribers. Here, we assume the users form an ad hoc network, and the communication is through direct links. Therefore

$$D_B = D_q \left(\frac{N' + K}{M} \right) + \max_{i \in N'} D_p(i) \quad (2)$$

where N' is the number of subscribers from whom the secondary buyers purchase the video stream and K is the number of subscribers within the coverage of the same base station, who are currently using the data service but cannot establish direct link to secondary buyers or are not willing to redistribute the video content. M is the maximal number of users that the network service provider can simultaneously afford. $D_q((N' + K)/M)$ is the network delay between subscribers and the service provider, and $D_p(i)$ is the processing time of subscriber i . From (2), we can see that, when the secondary buyer is purchasing the video from more subscribers, both the network delay and the processing delay of the video stream will be higher.

Without loss of generality, in this paper, we use the two-parameter rate-distortion model [28], which is widely employed in a medium-to-high bit rate situation, and the analysis for other models is similar. The two-parameter rate-distortion model is given as follows:

$$\text{Distortion} = \alpha e^{-\beta R} \quad (3)$$

where α and β are two positive parameters determined by the characteristics of the video content and R is the rate of the video.

Note that a secondary buyer is able to purchase the video from different subscribers in two different ways. Since the log and exponential functions are convex over \mathbb{R}^+ and the exponential functions are nondecreasing over \mathbb{R}^+ , it is easy to prove that buying different layers from different subscribers is a better choice since buying all layers from one subscriber is just a special case. The total bandwidth for the redistribution network is W , which is equably shared among the subscribers who are going to transmit. Hence, when the number of subscribers from whom the secondary user purchases, i.e., N' , increases, the bandwidth for transmitting each layer is smaller. Given the bit rate in (1), the mean square error of the video stream reconstructed by the secondary buyer B_j is

$$\begin{aligned} \text{MSE}_j &= \alpha \exp \left(-\beta \sum_{i \in \mathcal{N}'} R_{ij} \right) \\ &= \alpha \exp \left\{ -\beta \frac{W}{N'+1} \sum_{i \in \mathcal{N}'} \log_2 \left(1 + \frac{\text{SNR}_{ij}}{\gamma} \right) \right\}. \end{aligned} \quad (4)$$

If the subscribers do not share the bandwidth equally, then

$$\text{MSE}_j = \alpha \exp \left\{ -\beta \sum_{i \in \mathcal{N}'} W_i \log_2 \left(1 + \frac{\text{SNR}_{ij}}{\gamma} \right) \right\}$$

where W_i is the bandwidth allocated to subscriber i . Although we assume that the subscribers equally share bandwidth W , however, the following analysis can be applied to any $\{W_i\}_{i \in \mathcal{N}'}$.

III. OPTIMAL STRATEGIES FOR THE SINGLE-SECONDARY-BUYER CASE

In this section, we will first focus on the scenario where there is only one secondary buyer, i.e., $N_B = 1$. we will model the behavior of the subscribers and the secondary buyer as a Stackelburg game [29], [30] and then analyze and prove the existence of the equilibrium that leads to the optimal strategies for all users. When there is only one secondary buyer, we can remove superscript j for the secondary buyer index and have

$$\begin{aligned} \text{SNR}_i &= \frac{P_i H_i}{\sqrt{d_i} \sigma^2}, \quad R_i = \frac{W}{N'+1} \log_2 \left(1 + \frac{\text{SNR}_i}{\gamma} \right) \\ \text{MSE} &= \alpha \exp \left(-\beta \sum_i R_i \right) \\ &= \alpha \exp \left\{ -\beta \frac{W}{N'+1} \sum_i \log_2 \left(1 + \frac{\text{SNR}_i}{\gamma} \right) \right\}. \end{aligned} \quad (5)$$

A. Video-Stream Redistribution Game Formulation

Since the video-stream redistribution network is a dynamic system in which all users have high mobility that can join and leave anytime, it is very difficult to have a central authority to control the users' behavior. In addition, since this redistribution is unauthorized and illegal, to minimize their risk of being detected by the service provider, the participating users (subscribers and secondary buyers) have no incentives to trust one extra person and the central authority, and a distributed strategy is preferred.

Given the fact that there is only one secondary buyer, we propose a Stackelburg game model to analyze how the secondary buyer provide incentives for subscribers to redistribute the video stream and find the optimal price and quantity that the secondary buyer should offer. The ultimate goal of this analysis is to help the content owner to set an appropriate subscription fee such that the equilibrium of the game between the subscribers and the secondary buyers leads to negative payoffs. Thus, subscribers will have no incentive to redistribute the video.

Before the game starts, each user, either a subscriber or the secondary buyer, will declare his/her presence to all other users within his/her transmission range.

- *Game Stages:* The first stage of the game is the subscribers' (leaders') move. For each subscriber i , he/she will set his/her unit price p_i per unit transmission power, as well as his/her maximal transmission power $P_i^{(\max)}$. Then, in the second stage of the game, the secondary buyer (follower) will decide from whom to buy the video and how much power he/she wants the subscriber to transmit. The secondary buyer then pays each subscriber accordingly at the price that the subscriber sets in stage 1.
- *Utility function of the secondary buyer/follower:* We first define the secondary buyer's utility function and study his/her optimal action. The secondary buyer B gains rewards by successfully receiving the video with a certain quality. On the other hand, B has to pay for the power that the subscribers use for transmission. Let P_i be the power that the secondary buyer B decides to purchase from the i th subscriber S_i , the channel gain between S_i and B is H_i , and the distance between them is d_i . Therefore, given the video rate-distortion model, the utility function of the secondary buyer B_i can be defined as

$$\begin{aligned} \pi_B &= g_Q(\text{PSNR}_B - \text{PSNR}_{\max}) - g_D \left(D_B - D_q \left(\frac{K+1}{M} \right) \right) \\ &\quad - \left(\sum_i p_i P_i - p_o \right) \end{aligned} \quad (6)$$

where D_B is formulated as in (2), g_Q is a user-defined constant measuring the received reward if the PSNR of the reconstructed video is improved by 1 dB, and g_D is a constant measuring the user's loss if the video stream is further delayed by 1 s. PSNR_{\max} is the maximal PSNR of the video that can be obtained by subscribing to the service, and p_o is the price set by the content owner.

The aforementioned utility definition can be viewed as the difference between the utility if the secondary buyer buys the

video stream from the subscribers and the utility if he/she subscribes to the data plan. If the secondary buyer has subscribed to the data plan, then he/she will receive the video with the maximal PSNR, the delay of the video stream will only be the network delay, and the number of network users who are using the data service will be $K + 1$ in this case. The first term in (6) reflects the visual quality difference between the subscriber's video stream and the service provider's video stream. The second term considers the delay difference between the subscriber's video stream and the service provider's video stream. D_B was defined in (2), and $D_q((K + 1)/M)$ is the delay profile if the secondary buyer subscribes to the data plan and becomes an extra subscriber in the network. The third term indicates the price difference. The two constants g_Q and g_D control the balance between the gain and the loss of the secondary buyer. Since the service provider can always offer better video quality, i.e., $\text{PSNR}_{\max} \geq \text{PSNR}_B$ and $D_B \geq D_q(K + 1/M)$, all three terms of (6) are not positive. If the secondary buyer is very concerned about the video quality, i.e., g_Q and g_D are high, then (6) may be negative, and the secondary buyer will subscribe to the data plan himself/herself. Note that this redistribution behavior is not limited to video contents. For other digital contents, the reward terms of (6) will change according to the types of the content, but the payment term will remain the same.

- *Utility functions of the subscribers:* Each subscriber S_i can be viewed as a seller, who aims to earn the payment that covers his/her transmission cost and also to gain as much extra reward as possible. We introduce parameter c_i , i.e., the cost of power for relaying data, which is determined by the characteristics of the device that subscriber S_i uses. Hence, the utility of S_i can be defined as

$$\pi_{S_i} = (p_i - c_i)P_i \quad (7)$$

where P_i is the power that subscriber i uses to transmit to the secondary buyer. Thus, subscriber S_i will choose price p_i that maximizes his/her utility π_{S_i} .

The choice of the optimal price p_i is affected by not only the subscriber's own channel condition but also other subscribers' prices, since different subscribers noncooperatively play and they compete to be selected by the secondary buyer. Thus, a higher price may not help a subscriber improve his/her payoff.

B. Equilibrium Analysis

The aforementioned video-stream redistribution game is a game with perfect information, and the secondary buyer has perfect information of each subscriber's action (the selected price). According to backward induction [31], a game with perfect information has at least one equilibrium. Therefore, the optimal strategies for both the secondary buyer and the subscribers exist and can be obtained by solving the optimal decision for each stage using backward induction.

1) *Secondary Buyer's Optimal Strategy:* We analyze the game using backward induction and first study the secondary buyer's optimal strategy for a given price list from the subscribers. The secondary buyer B aims to determine the optimal power P_i that B should buy from each subscriber to maximize his/her own utility defined in (6).

Let L be the set including all subscribers who want to sell the video to the secondary buyer. Given that the secondary buyer purchases the transmission power P_i from subscriber S_i , the secondary buyer's received video rate is

$$R_B = \frac{W}{\sum_{i \in L} I[P_i > 0] + 1} \sum_{i \in L} \log_2 \left(1 + \frac{P_i H_i}{\gamma \sqrt{d_i} \sigma^2} \right) \quad (8)$$

where $I[\cdot]$ is the indicator function. Following the rate-distortion model in (5) and the transmission rate given in (5), the first term in (6) can be rewritten as a function of the transmission rate and is equal to

$$\begin{aligned} & g_Q(\text{PSNR}_B - \text{PSNR}_{\max}) \\ &= g_Q \left[10 \log_{10} \frac{255^2}{\alpha \exp(-\beta R_B)} - 10 \log_{10} \frac{255^2}{\alpha \exp(-\beta R_{\max})} \right] \\ &= g'_Q (R_B - R_{\max}) \end{aligned} \quad (9)$$

where $g'_Q = 10\beta g_Q \log_{10} e$ and R_{\max} is the video rate provided by the service provider.

Combining (5) and (6) with the aforementioned equation, we can formulate the utility function of B as a function of $\{P_i, \forall i \in L\}$. According to [27], the network delay of the 3G network is reciprocal to the network utilization percentage. Hence, the optimal strategy for the secondary buyer is

$$\begin{aligned} \max_{\{P_i\}} \quad & \pi_b = g'_Q (R_B - R_{\max}) - g_D \left(\max_{P_i > 0} D_p(i) + D_B \right) \\ & - \left(\sum_{i \in L} p_i P_i - p_o \right) \\ \text{s.t.} \quad & R_B \leq R_{\max} \quad P_i \leq P_i^{(\max)} \quad \forall i \in L \\ \text{where} \quad & D_B = \frac{MC}{M - K - \sum_{i \in L} I[P_i > 0]} - \frac{MC}{M - K - 1} \end{aligned} \quad (10)$$

and C is the network constant [27].

Note that, in (10) and (8), $\sum_{i \in L} I[P_i > 0]$ and $\max_{i \in L} D_p(i) I[P_i > 0]$ are piecewise continuous functions and are not necessarily continuous cross different sets of $S = \{i | P_i > 0\}$. Therefore, the optimization problem cannot be solved at once for the whole feasible set and has to be divided into subsets. Define subset $S_{N'}^{(k)}$ as the set with indexes of all the N' subscribers from whom the secondary buyer purchases the video stream and among whom subscriber k has the largest processing delay. Let $\mathbf{P} = [P_1, P_2, \dots, P_{N_s}]$ be the corresponding power vector, where P_i is the power that the secondary user purchases from subscriber i .

We can find the optimal power vector $\mathbf{P}_{N'}^{(k)}$ for subset $S_{N'}^{(k)}$ by making the first-order derivative of π_B with respect to P_i be zero, i.e.,

$$\frac{\partial \pi_B}{\partial P_i} = g'_Q \frac{W \ln 2}{N' + 1} \frac{A_i}{1 + A_i P_i} - p_i = 0 \quad \forall S_i \in L \quad (11)$$

where $A_i = \sqrt{d_i} \sigma^2 \gamma / H_i$. Therefore, if the secondary buyer purchases from any N' subscribers with the same maximal processing delay, then

$$P_i \left(S_{N'}^{(k)} \right) = \frac{g'_Q W \ln 2}{p_i (N' + 1)} - \frac{1}{A_i} \quad \forall S_i \in L \quad (12)$$

is the optimal solution. Note that (12) can be proved to be the unique maximizer for the subscriber set $\mathcal{S}_{N'}^{(k)}$ by finding the maximizer on the boundary. From (12), given the same maximal processing delay and the same number of subscribers from whom the secondary buyer is going to purchase, the second buyer purchases less from subscribers with higher prices. Also, the secondary buyer tends to purchase more from subscribers with better channels.

After the maximizer over each feasible subset is obtained, the secondary buyer should choose the one that gives himself/herself the largest utility. Let \mathbf{P}^* be the optimal decision of the secondary user; then, $\mathbf{P}_i^* = \arg \max_{0 \leq N', k \leq N_s} \pi_B(\mathbf{P}(\mathcal{S}_{N'}^{(k)}))$.

2) *Subscribers' Best Strategies*: The optimal price $p_i^*(\mathbf{H}_i, \mathbf{d}_i)$ should satisfy

$$\frac{\partial \pi_{S_i}}{\partial p_i} = P_i^* + (p_i - c_i) \frac{\partial P_i^*}{\partial p_i} = 0 \quad \text{s.t. } c_i \leq p_i \quad \forall i \in L \quad (13)$$

or be on the boundary, which means $p_i^* = c_i$. Given a set of subscribers $\mathcal{S}_{N'}^{(k)}$, this problem is a convex optimization problem, and the solutions can be numerically found. Note that the subscriber is willing to redistribute the video stream only if he/she can profit from the redistribution action. Therefore, a subscriber's claimed price should be higher than his/her cost.

C. Existence of the Equilibrium

In this section, we will prove that the optimal strategies of subscribers p_i^* in (13) and that of the secondary buyers \mathbf{P}^* in (12) form an equilibrium. By definition, if $(\{p_i^*\}, \mathbf{P}^*)$ is an equilibrium, then p_i^* is the best response of subscriber i if other subscribers choose $\{p_j^*\}_{j \neq i}$ and the secondary buyer chooses \mathbf{P}^* , and \mathbf{P}^* is the secondary buyer's best response if subscribers choose prices $\{p_i^*\}$.

Note that the optimization problem in (10) can be only done by dividing the problem into subproblems with different sets of subscribers from whom the secondary buyer actually purchases the video stream, i.e., $\mathcal{S}_{N'}^{(k)}$. Therefore, here, we first prove that, given any $\mathcal{S}_{N'}^{(k)}$, (12) and (13) form an equilibrium for the secondary buyer and all subscribers in $\mathcal{S}_{N'}^{(k)}$. Then, the actual equilibrium is the one that maximizes the secondary buyer's utility among these solutions.

For any given $\mathcal{S}_{N'}^{(k)}$, the optimization problem in (10) is equivalent to

$$\begin{aligned} \max_{P_i} \quad & \pi'_B = g'_Q R_B - \sum_{i \in \mathcal{S}_{N'}^{(k)}} p_i P_i, \\ \text{s.t.} \quad & R_B \leq R_{\max} \quad P_i \leq P_i^{(\max)} \quad \forall i \in \mathcal{S}_{N'}^{(k)} \\ \text{where} \quad & R_B = \frac{W}{k+1} \sum_{i \in \mathcal{S}_{N'}^{(k)}} \log_2 \left(1 + \frac{P_i H_i}{\gamma \sqrt{d_i} \sigma^2} \right). \end{aligned} \quad (14)$$

We first show that solution \mathbf{P}^* in (12) is the global optimum of (14) by showing the objective function in (14) being a concave function in P . The second-order derivatives of π'_B in (14) are

$$\frac{\partial^2 \pi'_B}{\partial P_i^2} = -\frac{g'_Q W \ln 2}{k+1} \frac{A_i^2}{(1 + A_i P_i)^2} < 0$$

$$\frac{\partial^2 \pi'_B}{\partial P_i \partial P_j} = 0$$

and

$$\begin{aligned} \frac{\partial^2 \pi'_B}{\partial P_i^2} \frac{\partial^2 \pi'_B}{\partial P_j^2} - \left(\frac{\partial^2 \pi'_B}{\partial P_i \partial P_j} \right)^2 \\ = \left(\frac{g'_Q W \ln 2}{k+1} \right)^2 \frac{A_i^2 A_j^2}{(1 + A_i P_i)^2 (1 + A_j P_j)^2} > 0. \end{aligned} \quad (15)$$

Moreover, π'_B is a continuous function of P_i . Thus, for $0 \leq P_i \leq P_{\max}$, π'_B is strictly concave in P_i and jointly concave over \mathbf{P} as well. Therefore, solution \mathbf{P}^* in (12) is the global optimum that maximizes the secondary buyer's utility π_B . Furthermore, in the real scenario, the secondary buyer can gradually increase power P_i for each subscriber to reach the optimal solution \mathbf{P}^* if there is information mismatch. For example, the knowledge of channel coefficients may change slowly, and the secondary buyer needs to adjust the strategy accordingly.

Then, we will show that, when other subscribers' prices are fixed, subscriber S_i cannot arbitrarily increase price p_i to get higher payoff. Given $\mathcal{S}_{N'}^{(k)}$, we take the first-order derivative of the optimal $P_i^*(\mathcal{S}_{N'}^{(k)})$ in (12) with respect to price p_i , i.e.,

$$\frac{\partial P_i^*(\mathcal{S}_{N'}^{(k)})}{\partial p_i} = -\frac{g'_Q W \ln 2}{k+1} \frac{1}{p_i^2} < 0 \quad (16)$$

which means that $P_i^*(\mathcal{S}_{N'}^{(k)})$ is a decreasing function of p_i . Such a phenomenon is reasonable since the secondary buyer tends to purchase less from subscribers with higher prices. Furthermore, when other subscribers' prices and the power that the secondary buyer purchases from each subscriber are fixed, the utility of subscriber i is a concave function of price p_i . The first-order derivative of subscriber i 's utility π_{S_i} with respect to price p_i is

$$\frac{\partial \pi_{S_i}}{\partial p_i} = \frac{g'_Q W \ln 2}{k+1} \frac{c_i}{p_i^2} > 0 \quad (17)$$

and we can also derive the second-order derivative of subscriber i 's utility π_{S_i} with respect to price p_i , i.e.,

$$\frac{\partial^2 \pi_{S_i}}{\partial p_i^2} = -\frac{2c_i g'_Q W \ln 2}{k+1} \frac{1}{p_i^3} < 0. \quad (18)$$

Therefore, π_{S_i} is concave with respect to price p_i . Due to the concavity of π_{S_i} , subscriber S_i can always find its optimal price p_i^* . As a result, $(\{p_i^*\}, \mathbf{P}^*)$ form an equilibrium.

IV. MULTIPLE SECONDARY BUYER CASE

In this section, we will extend the optimal strategy for the single-secondary-buyer case to the scenario with multiple secondary buyers.

A. Game Model

Assume that there are N_s subscribers and $N_b > 1$ secondary buyers. The first two stages of the game are the same as the single-secondary-buyer scenario, i.e., each subscriber declares the price per unit energy p_i , and then, each

secondary buyer B_j chooses the transmission power vector $\mathbf{P}^{(j)} = [P_1^{(j)}, P_2^{(j)}, \dots, P_{N_s}^{(j)}]$, where $P_i^{(j)}$ is the power that the secondary buyer j plans to purchase from subscriber i . With multiple secondary buyers, each subscriber i may receive several power purchase orders from different secondary buyers. In our paper, we let one subscriber transmit to one secondary buyer only. Thus, in the multiple-secondary-buyer scenario, the game model has an additional stage in which each subscriber i chooses the secondary buyer B_j who purchases the largest $P_i^{(j)}$ among all the N_b secondary buyers. Thus, for buyer B_j , the set of subscribers who will transmit video data to him/her is

$$N'_j = \{x | P_x^{(m)} < P_x^{(j)}, \forall m \neq j, m \in N_b\}. \quad (19)$$

Since each subscriber will answer to the secondary buyer who purchases the highest power, subscriber S_i 's utility function is

$$\pi_{S_i} = (p_i - c_i) \max_{j \in N_b} P_i^{(j)}. \quad (20)$$

For a secondary buyer B_j , his/her utility becomes

$$\pi_{B_j} = g'_Q(R_B - R_{\max}) - g_D \left(D_B + \max_{i \in N'_j, P_i^{(j)} > 0} D_p(i) \right) - \left(\sum_{i \in N'_j} p_i P_i^{(j)} - p_o \right) \quad (21)$$

where

$$R_B = \frac{W}{\sum_{i \in N_s, j \in N_b} I[P_i^{(j)} > 0] + 1} \sum_{i \in N'_j} \log_2 \left(1 + \frac{P_i^{(j)} H_{ij}}{\gamma \sqrt{d_{ij}} \sigma^2} \right). \quad (22)$$

B. Mixed-Strategy Equilibrium

Given the aforementioned definition of the utility functions, our next step is to find the subscribers and the secondary buyers' optimal decisions ($\{\mathbf{P}^{(j)*}\}_{j=1}^{N_b}, \{p_i^*\}_{i=1}^{N_s}$) from which no one in the system has the incentive to deviate.

Given the subscribers' price list $\{p_i\}$, for a secondary buyer B_j , the choice of the optimal power quantity $\mathbf{P}^{(j)*}$ is not only influenced by the channel conditions and the distances between subscribers and the secondary buyer B_j but also depends on the number of subscribers from whom B_j can purchase the video stream. For instance, if B_j is the only secondary buyer within the transmission range of S_i , B_j would always tend to use the optimal power in (12). If B_j has to compete with other secondary buyers, he/she might need to increase offer $P_i^{(j)}$ or switch to other subscribers.

The deterministic way to find the optimal strategy is to model the competition among secondary buyers as an auction problem. However, in a fast-changing mobile network, secondary buyers may not have enough interactions to learn how much others value the transmission power and the video bit stream. Also, without a central authorization, the final bided price may not be revealed to all subscribers. Instead, we focus on finding the optimal probability distribution over possible strategies and find the mixed-strategy equilibrium of the game. We will use backward induction to find the equilibrium. Backward induction first considers any decision that is made just before the end of the game, i.e., after each move stemming from this decision, the

game ends. If the player who makes such a decision rationally acts, he/she will choose the best move for himself/herself.

When using backward induction, given the subscribers price list for the unit transmission power $\{p_i\}_{i=1}^{N_s}$, the secondary buyer B_j chooses the probability function $f_j(\mathbf{P}^{(j)})$ that maximizes his/her own payoff, i.e.,

$$\max_{f_j(\mathbf{P}^{(j)})} E \left[\pi_{B_j} \left| \left\{ f_m(\mathbf{P}^{(m)}) \right\}_{m \in N_b, m \neq j} \right. \right] \quad (23)$$

where π_{B_j} is as defined in (21), or equivalently, each secondary buyer j seeks $f_j(\mathbf{P}^{(j)})$ that satisfies

$$E \left[\pi_{B_j}(\mathbf{P}^{(j)}) \left| \left\{ f_m(\mathbf{P}^{(m)}) \right\}_{m \in N_b, m \neq j} \right. \right] = C_j \quad \forall j \in N_b \quad (24)$$

where C_j is a constant [31].

We use an iterative best response algorithm for the secondary buyers to find the probability distribution $\{f_m(\mathbf{P}^{(m)})\}_{m \in N_b}$ below.

- 1) First, calculate the equilibrium power $\mathbf{P}^{(j)*}$ of each secondary buyer B_j based on (12) as a single secondary buyer. Also, let $f_j(\mathbf{P}^{(j)}) = \delta(\mathbf{P}^{(j)} - \mathbf{P}^{(j)*})$ for all $\mathbf{P}^{(j)} \neq \mathbf{P}^{(j)*}$.
- 2) For each $j \in N_b$, given $\{f_m(\mathbf{P}^{(m)})\}_{m \in N_b, m \neq j}$, solve (24), and update $f_j(\mathbf{P}^{(j)})$.
- 3) Repeat the aforementioned step until the solutions converge.

We will show the convergence of the aforementioned algorithm by simulations in the next section. After solving $\mathbf{P}^{(j)*}$ for a given the price vector $\{p_i\}_{i=1}^{N_s}$, the optimal pricing $\{p_i^*\}_{i=1}^{N_s}$ can be similarly calculated by exhaustive search.

C. Simulation Results

In this section, we will show the equilibrium of the video-stream redistribution game under different scenarios, as well as the optimal price for the content owner.

1) *Single Secondary Buyer*: In our simulations, the secondary buyer is located at the origin (0,0), and the subscribers are initially uniformly distributed in a rectangle of size 100 m by 100 m centered around the origin. The pricing game is played 100 times, and each subscriber changes its location each time the game restarts. For each subscriber, the location change is normally distributed with zero mean and unit variance. The direction of each subscriber's location change follows the uniform distribution. For all users, the maximal transmit power P_{\max} is 100 mW, and the noise level is 10^{-2} W. The capacity gap is $\gamma = 1$, the total available bandwidth is $W = 300$ kHz, $g_Q = 0.1$, and $g_D = 0.1/\text{ms}$, and for subscriber S_i , his/her cost per unit transmission power c_i is a random variable following uniform distribution in the range [0.05, 0.15]. The processing delay of each subscriber $D_p(i)$ is also a uniformly distributed random variable in [0.1, 10.1] ms. We use the video sequence "Akiyo" in Quarter Common Intermediate Format (QCIF) and then encode it based on H.264 JM 9.0. The resulted rate-distortion parameter $\beta = 0.0416$, and $\alpha = 6.8449$. We set the maximal PSNR, which is provided by the original content owner, as 35 dB, and the corresponding maximal bit rate for Akiyo is $R_{\max} = 84$ kb/s. For simplicity and without loss of generality, the subscription price p_o for the video sequence is set to be 0 so that the optimal price for the content owner can be simply viewed as $-\pi_B$. It implies that, if the secondary buyer's

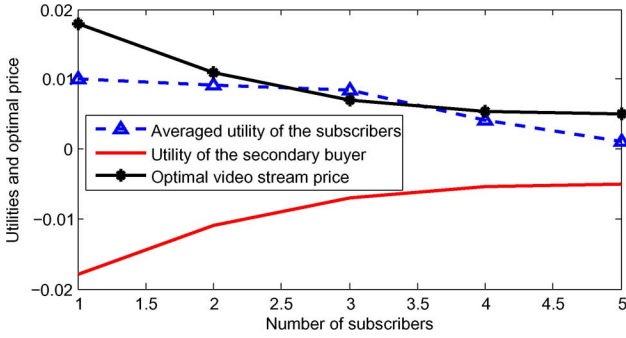


Fig. 2. Utilities of the users and the optimal video-stream price versus different number of subscribers.

utility is negative, then he/she has incentives to purchase the video stream from the content owner.

First, we let $M - K = 50$, and K , which is the number of subscribers who are currently using the data service but are not in the redistribution network, varies from 1 to 5, i.e., the network is not crowded and the number of subscribers is small when compared with the maximal number that the network can afford. In Fig. 2, we observe that, as the number of available subscribers increases, the competition among subscribers becomes more severe; thus, the optimal price for the content owner decreases. When there are no more than three subscribers, the averaged utility of the subscribers does not vary much. This is because, in such cases, the secondary buyer is trying to purchase the maximum video rate R_{\max} from all subscribers to increase the reconstructed video's quality, and these subscribers are not competing with each other. However, when there are more subscribers, the secondary buyer can easily get the video quality close to PSNR_{\max} , and subscribers compete with each other to motivate the secondary buyer to purchase from himself/herself. Such a phenomenon is the nature of free market with more sellers.

Next, we will examine the impact of network quality on the optimal price of the video stream. From Fig. 2, we can see that the competition among subscribers dominates their own utilities, and the optimal price for the video stream (at the content owner's side) does not vary much when there are more than three subscribers. Therefore, here, we set the total number of subscribers as three, and Fig. 3 shows how the video bit rate R_{\max} and the network usage influence the optimal video-stream price. In Fig. 3, $M - K$ varies from 5 to 50, and a smaller $M - K$ means that the current number of users in the network is approaching the network capacity and the network is more crowded. We select $R_{\max} = 75, 84, \text{ and } 89$ kb/s, and the resulting maximum PSNRs are 30, 35, and 40 dB, respectively. From Fig. 3, we can see that, if the service provider can offer a good video quality that the redistribution network cannot achieve, he/she can charge more for the streaming service. Also, when the network is very busy with a smaller $M - K$, the video delay dominates the video quality, and therefore, the secondary buyer tends to purchase from a smaller number of subscribers, but each subscriber can only provide limited video quality. Hence, for the content owner, providing a better quality streaming service is critical when the network is busy, and the content owner has to maintain the network quality at a level

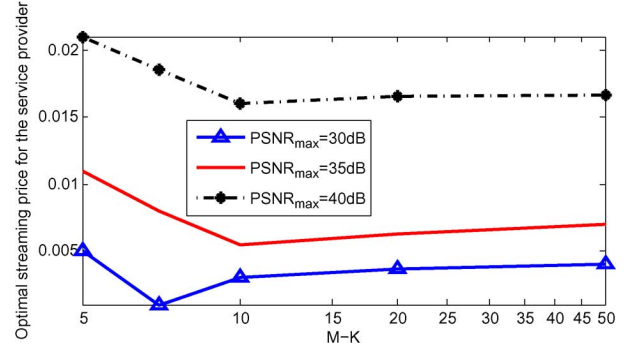


Fig. 3. Optimal video-stream price versus qualities of network and streaming service.

that the quality of the video stream is not sacrificed. Furthermore, for a fixed R_{\max} , when the network delay is small enough, e.g., $M - K$ is less than 10 for $\text{PSNR} = 40$ dB, the data service price starts to degrade as $M - K$ increases. This is because the secondary buyer purchases the video from the redistribution network. However, when the network delay is relatively small, buying from more subscribers will not introduce too much relay, and secondary buyers will be willing to purchase from enough number of subscribers to reach the highest data rate R_{\max} and the maximum possible PSNR_{\max} . However, compared with purchasing from the content owner, although the video rate is the same, delay will be slightly larger. For instance, when $M - K$ is larger than 20 and $\text{PSNR} = 30$ dB, the optimal data service price starts to get slightly higher since buying from subscribers introduces a slightly larger delay.

2) *Multiple Secondary Buyers*: We consider the system setup where secondary buyers are uniformly distributed in a 100 m by 100 m square centered around the origin. There are three subscribers located at (25,10), (25, -10), and (0, -30), respectively. Other system parameters are the same as in the previous section.

In Fig. 4, we observe that, as the total number of secondary buyers increases, the competitions among the secondary buyers become more severe, and the optimal price for the content owner increases. When there are fewer than three secondary buyers, the averaged utility of the secondary buyers does not vary much since each secondary buyer has a high probability to receive the video from at least one subscriber. Comparing the utilities of the subscribers and that of the secondary buyers when there are more than three secondary buyers, it is clear that the increment in the subscribers' utilities is much smaller than the decrease in the secondary buyers' utilities. Such a phenomenon is because secondary buyers compete with each other and some secondary buyers may not even receive anything from the subscribers.

Fig. 5 shows the convergence speed of the iterated algorithm to find the mixed strategy equilibria. It is clear that the algorithm converges. With more users in the network, the algorithm takes more iterations to find the equilibria.

Table I shows the optimal pricing under different simulation setting. An example of single secondary buyer with three subscribers is shown in Table I(a). From Table I(a), we can see that the competition among the subscribers results in equivalent equilibrium price and that the secondary buyer is purchasing an

TABLE I
EQUILIBRIUM PRICE AND RATE OFFERED BY EACH SUBSCRIBER. (A) SINGLE SECONDARY BUYER. (B) THREE SECONDARY BUYERS

| | Subscriber 1 | Subscriber 2 | Subscriber 3 |
|-----------|--------------|--------------|--------------|
| Price | 0.98 | 0.11 | 0.98 |
| Rate(bps) | 42 | 0 | 42 |
| Cost | 0.072 | 0.11 | 0.091 |
| Location | (39, 17) | (-19, 21) | (-4, 27) |

| | Subscriber 1 | Subscriber 2 | Subscriber 3 |
|-----------|--------------|--------------|--------------|
| Price | 0.12 | 0.12 | 0.14 |
| Rate(bps) | 84 | 84 | 84 |
| Cost | 0.072 | 0.11 | 0.091 |
| Location | (0,-30) | (25,-10) | (25,10) |
| Buyer | buyer 3 | buyer 2 | buyer 1 |
| | buyer 1 | buyer 2 | buyer 3 |
| Location | (-17, 18) | (39, -41) | (-7, -23) |

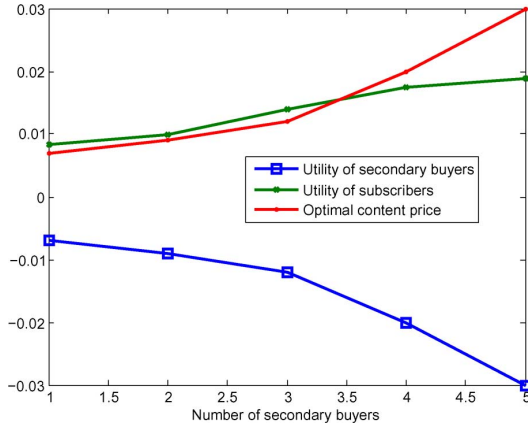


Fig. 4. Utilities of the users and the optimal video-stream price versus different number of secondary buyers with three subscribers.

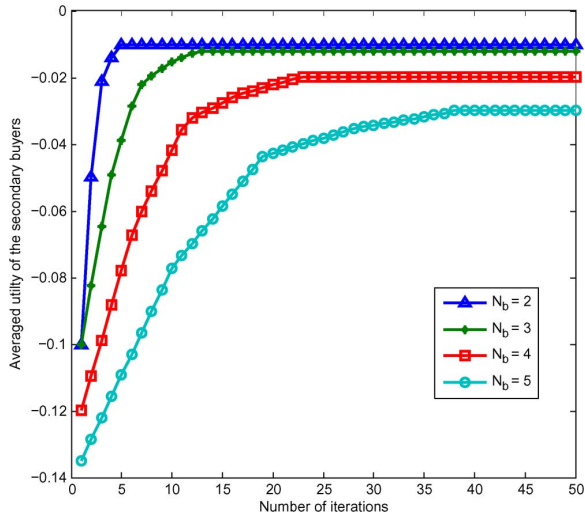


Fig. 5. Convergence of the iterated algorithm.

even amount of data from the subscribers. However, for subscriber 2 whose transmission cost is significantly higher than other subscribers, he/she has to set the price that is the same as his/her cost and gain zero utility. Table I(b) shows an example of three secondary buyers and three subscribers with the same cost parameter as in Table I(a). Note that the equilibrium price set by subscriber 3 is higher than that of the other two subscribers because buyer 1 is located so far away from the other two subscribers that subscriber 3 is buyer 1's only choice if buyer 1 wants to receive the highest-PSNR video. In both examples, the

parameters (distance between users, bandwidth, number of secondary buyers, and maximal power) allow all secondary buyers to receive the same-quality video stream from the subscribers so that the dominating factor of pricing is the competition among users.

V. OPTIMAL PRICING FOR THE CONTENT OWNER

In the previous sections, we have discussed the equilibria and the optimal pricing strategy in the video redistribution network. Our assumption there is that the content owner would like to set price p_o smaller than the equilibrium price in the redistribution network. By doing so, the secondary buyers would have no incentives to purchase the video content from the subscribers and will always subscribe to the data plan from the service provider. However, such a strategy may not always maximize his/her total income, i.e., the price times the number of subscribers. In this section, we consider the scenario where the service provider's goal is not the prevention of video redistribution but rather the maximization of his/her own income. We include the service provider as a player in the game and find his/her optimal strategies.

A. Pricing Game Model and Evolution Dynamics

Here, we model the video pricing problem for the content owner as a noncooperative game, which can be played several times. For example, in practical scenarios, the service provider can always change the price if the total income is below the expectation. Also, even when the price is fixed, mobile users can change their mind on whether to subscribe to the data plan or to purchase from other subscribers. Such natural repetitions help the players find the equilibrium.

The basic elements of the game are listed below.

- **Game Stages:** In the video pricing game, the first mover is the service provider, who first sets the price of the video content p_o . Then, N_a mobile users who are interested in the video content decide whether to subscribe to the video streaming service. Since, based on the analysis in Sections III and IV, the redistribution of the video content is possible, mobile users also take into consideration the possible payoffs that they can get in the redistribution network when making the decision.
- **Utility function of the service provider:** Obviously, the content owner's utility is the price times the number of subscribers, i.e.,

$$\pi_c = p_o \times N_s \quad (25)$$

where N_s is the number of subscribers. With a higher price, there will be fewer subscribers and a smaller N_s value, particularly when it is possible for mobile users to receive the video content from the redistribution network. Therefore, the service provider cannot arbitrarily increase the data service price p_o and has to consider mobile users' utilities.

- **Utility function of the mobile users:** Each mobile user in N_a has the choice to pay p_o to subscribe to the data plan or to purchase the content from other subscribers instead. Assume that, among the N_a mobile users, $N_s \geq 0$ of them subscribe to the data plan, and the rest of the $N_b = N_a - N_s \geq 0$ users decide not to. Let $\pi_s(N_s, N_b)$ and $\pi_B(N_s, N_b)$ be the utilities that a subscriber and a secondary buyer can get from the redistribution network as in Section IV, respectively.

If user i decides to subscribe to the data plan, then his/her utility contains two parts. The first part is from the subscription to the streaming service, where he/she enjoys the video content with higher quality and shorter delay at a cost of the subscription fee. The second part is from the redistribution of the video to secondary buyers. Hence, if user i chooses to become a subscriber, his/her utility is

$$\pi_i(s, N_s, N_b) = \pi_s(N_s, N_b) + g_{Qi} * \text{PSNR}_{\max} - g_{Di} D_q \left(\frac{K+1}{M} \right) - p_o \quad (26)$$

where g_{Qi} is user i 's gain-per-decibel improvement in the PSNR of the reconstructed video and g_{Di} is user i 's cost-per-second delay in receiving the bit stream. The first input parameter s in $\pi_i(s, N_s, N_b)$ denotes the action "subscribe." Note that $\pi_s(N_s, N_b) = 0$ if N_s or N_b is equal to 0.

If user i chooses not to subscribe to the data plan, his/her utility only comes from the redistribution network by purchasing the video content from subscribers, i.e., he/she gains nothing from the service provider and also pays nothing to the service provider. Hence

$$\begin{aligned} \pi_i(n, N_s, N_b) &= g_Q \text{PSNR}_B - g_D D_B - \sum_{j \in N'_i} p_j P_j^{(i)} \\ &= \pi_B(N_s, N_b) + g_{Qi} * \text{PSNR}_{\max} \\ &\quad - g_{Di} D_q \left(\frac{K+1}{M} \right) - p_o \end{aligned} \quad (27)$$

where the first input "n" in $\pi_i(n, N_s, N_b)$ denotes the action "do not subscribe." Note that $\pi_i(n, N_s, N_b) = 0$ if N_s or N_b is equal to 0.

To analyze this game, we first investigate the equilibrium strategy of the mobile users given the data service price p_o . As mentioned before, the previously mentioned pricing game can be repeatedly played, and mobile users may use their previous experience to adjust their strategies accordingly. Therefore, a stable strategy for all mobile users that is robust to mutants of users' strategies is preferred in the pricing game. To find the stable equilibrium, we will use the evolutionary game theory to analyze the evolution of the mobile users' behavior and to derive the evolutionarily stable equilibrium, which leads to the optimal price of the video content.

The *evolutionarily stable equilibrium*, inspired by biology mutations, guarantees the stability of the outcome of the game. The evolutionary game theory focuses on the dynamics of strategy change more than the properties of strategy equilibria, which is the core of the traditional game theory. The evolutionarily stable equilibrium provides guidance for a rational player to approach the best strategy against a small number of players who deviate from the best strategy and thus achieve stability. The evolutionarily stable equilibrium is defined below.

Definition 1: An ESS is action a^* in the strategy space A such that:

- equilibrium condition: $\pi_i(a, a^*) \leq \pi_i(a^*, a^*)$;
- stability condition: if $\pi_i(a, a^*) = \pi_i(a^*, a^*)$, $\pi_i(a, a) < \pi_i(a^*, a)$ for every best response $a \neq a^* \in A$.

Since each mobile user is not certain of other users' decisions, he/she may try different strategies in every play and learn from the interactions. For example, a mobile user may try to change from "subscribe" to "do not subscribe" and observe whether his/her utility received from the redistribution network is satisfactory. During such a learning process, the percentage, i.e., the population share, of players using a certain pure strategy ("subscribe" or "do not subscribe") may change. The stable percentage of mobile users that chooses to subscribe to the data plan is what we are interested in.

The population evolution can be characterized by replicator dynamics as follows: at time t , let $N_s(t)$ denote the number of mobile users that subscribe to the data plan, then the subscribers' population state $x_s(t)$ is defined as

$$x_s(t) = \frac{N_s(t)}{N_a} \quad (28)$$

and $x_b(t) = N_b(t)/N_a = 1 - x_s(t)$ is the secondary buyers' population state. By replicator dynamics, the evolution dynamics of $x_s(t)$ at time t is given by the following differential equation:

$$\dot{x}_s = \eta [\bar{\pi}_s(x_s) - \bar{\pi}(x_s)] x_s \quad (29)$$

where \dot{x}_s is the first-order derivative of $x_s(t)$ with respect to time t , $\bar{\pi}_s(x_s)$ is the average payoff of mobile users who subscribe to the data plan, and $\bar{\pi}(x_s)$ is the average payoff of all mobile users. η is a positive scale factor. We can see that, if subscribing to the data plan can lead to a higher payoff than the average level, the probability of a user switching to "subscribe" will grow and the growth rate \dot{x}_s/x_s is proportional to the difference between the average payoff of subscribers $\bar{\pi}_s(x_s)$ and the average payoff of all mobile users $\bar{\pi}(x_s)$. The other intuition behind x_s is that x_s can be viewed as the probability that one mobile user adopts pure strategy "subscribe," and the population state vector $\mathbf{x} = \{x_s(t), x_b(t)\}$ is equivalent to a mixed strategy for that player. If subscribing to the data plan results in a higher payoff than the mixed strategy, then the probability of subscribing to the data plan should be higher, and x_s will increase. The rate of the increment is proportional to the difference between the payoff of adopting the pure strategy "subscribe" and the payoff achieved by using the mixed strategy $\mathbf{x} = \{x_s, x_b\}$.

Given the evolution dynamics previously formulated, in the following sections, we will derive the evolutionarily stable equilibrium among mobile users in different scenarios.

B. Analysis of Pricing Game With Homogeneous Mobile Users

A strategy is an evolutionarily stable equilibrium (ESS) if and only if it is asymptotically stable to the replicator dynamics. In the pricing game, when time goes to infinity, if (29) is equal to zero, then \mathbf{x} is the evolutionarily stable equilibrium. In this section, we first focus on the scenario where all mobile users value the video equality in the same way and $g_{Qi} = g_{Qj} = g_Q$ and $g_{Di} = g_{Dj} = g_D$ for all $i, j \in N_a$. The scenario in which different mobile users have different values of the video quality will be analyzed in the next section.

Let $Q = g_Q * \text{PSNR}_{\max} - g_D D_a ((K + 1)/M) - p_o$, then in the homogeneous case, the utilities of the subscribers and the secondary buyers are

$$\begin{aligned} \pi(s, N_s, N_b) &= \pi_s(N_s, N_b) + Q \\ \pi(n, N_s, N_b) &= \pi_B(N_s, N_b) + Q \end{aligned} \quad (30)$$

respectively. Note that mobile users are homogeneous and they will have the same evolution dynamics and equilibrium strategy. Given that x_s is the probability that a mobile user decides to subscribe to the data plan, the averaged utilities of the subscribers and the secondary users are

$$\begin{aligned} \bar{\pi}_s(x_s) &= \sum_{i=0}^{N_a} \binom{N_a}{i} x_s^i (1 - x_s)^{N_a - i} \pi(s, i, N_a - i) \\ \bar{\pi}_b(x_s) &= \sum_{i=0}^{N_a} \binom{N_a}{i} x_s^i (1 - x_s)^{N_a - i} \pi(n, i, N_a - i) \end{aligned} \quad (31)$$

respectively. The average utility of all mobile users is

$$\bar{\pi}(x_s) = x_s \cdot \bar{\pi}_s(x_s) + x_b \cdot \bar{\pi}_b(x_s). \quad (32)$$

Given the aforementioned equation, (29) can be rewritten as

$$\dot{x}_s = \eta [\bar{\pi}_s(x_s) - \bar{\pi}(x_s)] x_s = \eta [\bar{\pi}_s(x_s) - \bar{\pi}_b(x_s)] x_b x_s. \quad (33)$$

In equilibrium x_s^* , no player will deviate from the optimal strategy, indicating $\dot{x}_s = 0$ in (33). We can then obtain the equilibria, which are $x_s = 0$, $x_s = 1$ or $x_s = \bar{\pi}_s(x_s) - \bar{\pi}_b(x_s)$. To verify that they are indeed ESS, we will show that these three equilibria are asymptotically stable, i.e., the replicator dynamics (29) converges to these equilibrium points.

The first step is to guarantee that $x_s(t) + x_n(t) = 1$ for all t , which means that the sum of the probabilities of a mobile user subscribing to the data plan is not equal to one. We can verify it by summing up (29) with the reciprocal dynamic function of x_n , which is

$$\dot{x}_b = \eta [\bar{\pi}_b(x_b) - \bar{\pi}(x_s)] x_b. \quad (34)$$

Combining (29) and (34), we have

$$\dot{x}_b + \dot{x}_s = \eta [x_s \bar{\pi}_s(x_s) + x_b \bar{\pi}_b(x_b) - (x_s + x_b) \bar{\pi}(x_s)] = 0. \quad (35)$$

Recalling that $\bar{\pi}(x_s) = x_s \times \bar{\pi}_s(x_s) + (x_n) \times \bar{\pi}_b(x_s)$ and $x_s(0) + x_n(0) = 1$, the aforementioned equation is equivalent to $\dot{x}_n + \dot{x}_s = 0$. As a result, $x_s(t) + x_n(t) = x_s(0) + x_n(0) = 1$ for all t in the evolution process.

Next, we need to show that all the nonequilibrium strategies of the pricing game will be eliminated during the evolution process. If the replicator dynamics is a myopic adjustment

dynamic, then all nonequilibrium strategies will be eliminated during the process. A dynamic is myopic adjustment if and only if

$$\sum_{a \in A} \dot{x}_a \bar{\pi}(x_a, x_{-a}) \geq 0 \quad (36)$$

where A is the strategy space, x_a is the population of users adopting pure strategy a , and $\bar{\pi}(x_a, x_{-a})$ is the average payoff of users adopting pure strategy a . For our optimal pricing game, the strategy space is $A = \{s, b\}$, where “ s ” means “subscribe” and “ b ” means “do not subscribe” and be a secondary buyer. Combining (29) with (34) and (32), we have

$$\begin{aligned} & \sum_{a \in \{s, b\}} \dot{x}_a \bar{\pi}_a(x_a) \\ &= \sum_{a \in \{s, b\}} \eta [\bar{\pi}_a(x_a) - \bar{\pi}(x_s)] x_a \bar{\pi}_a(x_a) \\ &= \sum_{a \in \{s, b\}} \eta \left(\bar{\pi}_a(x_a) - \sum_{a' \in \{s, b\}} x_{a'} \bar{\pi}_{a'}(x_{a'}) \right) x_a \bar{\pi}_a(x_a) \\ &= \eta \left\{ \sum_{a \in \{s, b\}} x_a \bar{\pi}_a^2(x_a) - \left[\sum_{a \in \{s, b\}} x_a \bar{\pi}_a(x_a) \right]^2 \right\} \geq 0. \end{aligned} \quad (37)$$

In (37), the last inequality is from the Jensen inequality, which says $(a_1 x_1 + a_2 x_2)^2 \leq a_1 x_1^2 + a_2 x_2^2$ with $a_1 + a_2 = 1$ and x^2 being a concave function of x . Therefore, the reciprocal dynamics of the pricing game in (29) is myopic adjustment and will eliminate all nonequilibrium strategies.

From (33), \dot{x}_s has the same sign as $\bar{\pi}_s(x_s) - \bar{\pi}_b(x_s)$. According to the discussions in Sections III and IV, $\bar{\pi}_s(x_s)$ is a decreasing function of x_s , whereas $\bar{\pi}_b(x_s)$ is an increasing function of x_s . Therefore, when x_s goes from 0 to 1, the sign of \dot{x}_s either does not change or changes only once.

- 1) When $\bar{\pi}_s(x_s) > \bar{\pi}_b(x_s)$ for all $x_s \in [0, 1]$, in the evolution process, $\dot{x}_s = dx_s(t)/dt > 0$ for all t , and (29) converges to $x_s = 1$, which is an ESS.
- 2) If $\bar{\pi}_s(x_s) < \bar{\pi}_b(x_s)$ for all $x_s \in [0, 1]$, in the evolution process, $\dot{x}_s = dx_s(t)/dt < 0$ for all t , and (29) converges to $x_s = 0$, which is the ESS in this scenario.
- 3) When $\bar{\pi}_s(x_s) - \bar{\pi}_b(x_s) = 0$ has one and only one root x_s^* , and (29) converges to ESS x_s^* .

Therefore, for each price p_o set by the content owner, we can find the stable number of subscribers $N_s = N_a \cdot x_s^*$, from which we can calculate the service provider’s utility. Hence, given the ESS of the mobile users, by backward induction, the service provider can easily choose the optimal data service price to maximize his/her own payoff.

C. Analysis of Pricing Game With Heterogeneous Mobile Users

In the heterogeneous scenario where different mobile users value video quality differently, it is very difficult to represent the average payoff of the subscribers and that of the secondary buyers in a compact form. Hence, we start with the simple two-person game and find its ESS. We then extend the ESS into the scenario with multiple heterogeneous mobile users.

TABLE II
MATRIX FORM OF THE PRICING GAME WITH TWO HETEROGENEOUS MOBILE USERS

| | Subscribe | Do not subscribe |
|------------------|------------------------------|------------------------------|
| Subscribe | (Q_1, Q_2) | $(Q_1 + \pi_{1s}, \pi_{2b})$ |
| Do not subscribe | $(\pi_{1b}, Q_2 + \pi_{2s})$ | $(0, 0)$ |

We first start with the two-player game. Assume that there are two mobile users with different $\{g_{Qi}, g_{Di}, c_i\}$. If both of them decide not to subscribe to the data plan, then they pay nothing and gain nothing from the service provider. Also, since there are no subscribers, the redistribution network does not exist, and both players' utilities are 0. If both decide to subscribe to the data plan, then the redistribution network also does not exist either, since there are no secondary buyers. In this scenario, the utilities of player i is $Q_i = g_{Qi} * \text{PSNR}_{\max} - g_{Di}D_q((K + 1)/M) - p_o$. If player 1 becomes a subscriber but player 2 decides not to subscribe, then player 1's utility is $Q_1 + \pi_{1s}$, and player 2's utility is π_{2b} . Here, π_{1s} and π_{2b} are the utilities that users 1 and 2 get from the redistribution network as a seller and a buyer, respectively, and their calculation is the same as that in Sections III and V. When only player 2 subscribes to the streaming service, the analysis is similar, and we can obtain the matrix form of the game shown in Table II. In Table II, each row represents user 1's decision, and each column represents user 2's decision. For each entry in the table, the first term is user 1's payoff, and the second term is user 2's payoff.

Let x_1 and x_2 be players 1 and 2's probability of adopting the pure strategy "subscribe," respectively. Then, the expected payoff $\bar{\pi}_1(s)$ of user 1 by always playing "subscribe" is

$$\bar{\pi}_1(s, x_2) = Q_1x_2 + [Q_1 + \pi_{1s}](1 - x_2) \quad (38)$$

and the expected payoff of player 1 when he plays the mixed strategy x_1 is

$$\bar{\pi}_1(\mathbf{x}) = Q_1x_1x_2 + [Q_1 + \pi_{1s}]x_1(1 - x_2) + \pi_{1b}(1 - x_1)x_2. \quad (39)$$

Then, we can write the reciprocal dynamics of x_1 as

$$\dot{x}_1 = x_1(1 - x_1)[(Q_1 + \pi_{1s}) - (\pi_{1s} + \pi_{1b})x_2] \quad (40)$$

and similarly

$$\dot{x}_2 = x_2(1 - x_2)[(Q_2 + \pi_{2s}) - (\pi_{2s} + \pi_{2b})x_1]. \quad (41)$$

An equilibrium point must satisfy $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$; then, from (40) and (41), we get five equilibria $(0, 0)$, $(0, 1)$, $(1, 0)$, $(1, 1)$, and $((Q_2 + \pi_{2s})/(\pi_{2s} + \pi_{2b}), (Q_1 + \pi_{1s})/(\pi_{1s} + \pi_{1b}))$.

If we view (41) and (40) as a nonlinear dynamic system, then the aforementioned five equilibria are ESSs if they are locally asymptotically stable. The asymptotical stability requires that the determination of the Jacobian matrix J be positive and the trace of J be negative. The Jacobian matrix J can be derived by taking the first-order partial derivatives of (41) and (40) with respect to x_1 and x_2 , and

$$J = \begin{bmatrix} (1 - 2x_1)D_1 & -x_1(1 - x_1)(\pi_{1s} + \pi_{1b}) \\ -x_2(1 - x_2)(\pi_{2s} + \pi_{2b}) & (1 - 2x_2)D_2 \end{bmatrix} \quad (42)$$

where $D_1 = (Q_1 + \pi_{1s}) - (\pi_{1s} + \pi_{1b})x_2$ and $D_2 = (Q_2 + \pi_{2s}) - (\pi_{2s} + \pi_{2b})x_1$. By jointly solving $\det(J) > 0$ and $\text{trace}(J) < 0$, we can have the optimal subscription strategies for mobile users under different scenarios.

- 1) When $Q_1 + \pi_{1s} < 0$ and $Q_2 + \pi_{2s} < 0$, there is one ESS $(0, 0)$, and both users tend to not subscribe to the data plan.
- 2) When $Q_1 - \pi_{1b} < -(Q_2 + \pi_{2s})$ and $(Q_2 + \pi_{2s})(Q_1 - \pi_{1b}) < 0$, there is one ESS $(0, 1)$, and the strategy profile users 1 and 2 adopt converges to (not subscribe, subscribe).
- 3) When $Q_1 - \pi_{1b} < -(Q_1 + \pi_{1s})$ and $(Q_1 + \pi_{1s})(Q_2 - \pi_{2b}) < 0$, there is one ESS $(1, 0)$, and user 1 tends to subscribe, while user 2 tends to not subscribe to the data plan.
- 4) When $Q_1 - \pi_{1b} > 0$ and $Q_2 - \pi_{2b} > 0$, there is one ESS $(1, 1)$, and both users tend to subscribe to the data plan.

We can see that, when Q_1 is higher with larger g_{Q1} and g_{D1} , user 1 tends to subscribe to the data plan.

Based on the aforementioned discussion on the ESSs of the two-player game, we can infer that the users who value the video quality more (with higher g_{Qi} and g_{Di}) would intend to subscribe to the data plan. Users with smaller g_{Qi} and g_{Di} would tend to choose "do not subscribe" and become secondary buyers. However, if the data service price p_o is too high so that the subscription gives all users negative payoff, no player would subscribe to the service.

D. Simulation Results

Here, we will verify the derived ESS and show by simulation results the optimal price for the content owner if he/she wants to maximize his/her utility. We first test on the homogeneous scenario that there are six mobile users who are initially uniformly located in a 100 m by 100 m square centered around the origin. All six mobile users have the same gain weighting factors $g_Q = 0.1$ and $g_D = 0.1/\text{ms}$. The pricing game is played 100 times, and each secondary buyer changes its location after the game restarts. The distance between each secondary buyer's locations in two consecutive games is normally distributed with zero mean and unit variance. The direction of each secondary buyer's location change follows the uniform distribution. Other simulation settings are the same as in Section IV. We use the video sequence "Akiyo" in QCIF as in the single-secondary-buyer scenario. The mobile users changes their strategies and evolve according to (29).

Fig. 6 shows the content owner's utility when the PSNRs of the video stream are 30 and 40 dB, respectively. $M - K$ reflects how crowded the mobile network is. It is clear that, if the content owner provides better quality network or video, its payoff can be increased. Also, for lower quality videos, the content owner's utility saturates earlier than high-quality videos with respect to the network quality, which means that, if the content owner decides to offer low-quality videos, to maximize its utility, it tends to offer low-quality network as well.

Furthermore, in Fig. 7, we show the utility of the service provider with heterogeneous mobile users with different evaluations of video quality. Similar to the settings in Fig. 6, there are six mobile users who are initially uniformly located in a 100 m by 100 m square centered around the origin. In Fig. 7(a), all six mobile users' gain weighting factor for video quality $g_Q = 0.1$, but two of them have a delay gain factor $g_D = 0.1$,

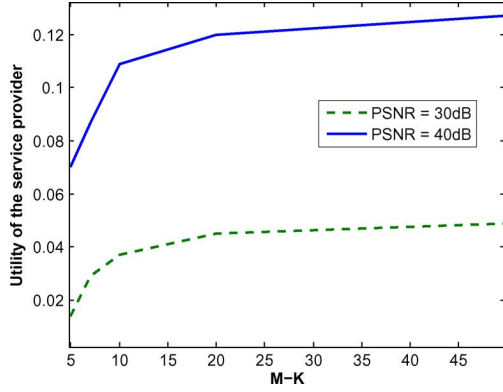


Fig. 6. Utilities of the service provider versus network quality with homogeneous mobile users.

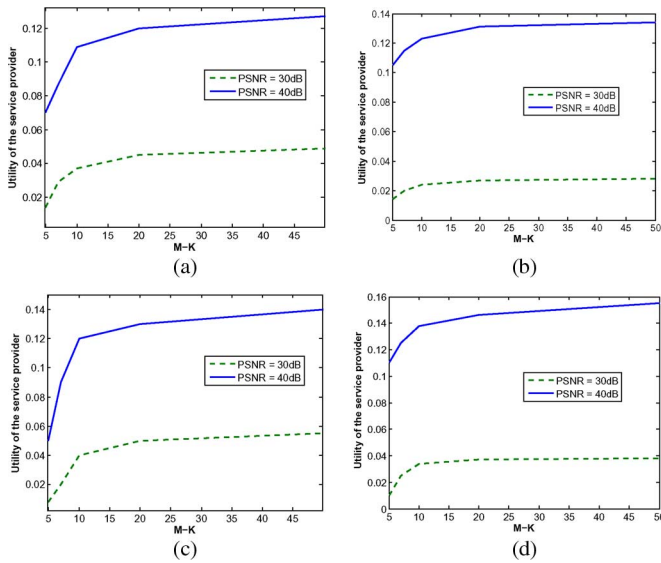


Fig. 7. Utilities of the service provider versus network and video quality with heterogeneous mobile users. (a) $g_Q = 0.1$; $g_D = 0.1, 0.15, 0.2$. (b) $g_Q = 0.1, 0.15, 0.2$; $g_D = 0.1$. (c) $g_Q = 0.2$; $g_D = 0.15, 0.2, 0.25$. (d) $g_Q = 0.15, 0.2, 0.25$; $g_D = 0.2$.

two of them have a delay factor $g_D = 0.15$, and the rest two of them are with a delay factor $g_D = 0.2$. From Figs. 6 and 7, we can clearly see that providing higher quality network service (larger $M - K$) gives the content owner a lot more gain than providing low-quality network service. Also, the utility difference between providing low (PSNR = 30 dB) and high (PSNR = 40 dB) video qualities is lesser in Fig. 7(a) than in Fig. 6. Furthermore, comparing the service provider's utility when $M - K = 45$ with the case that $M - K = 5$ in Figs. 6 and 7(a), we can see that providing better network quality gives the service provider more gain in the scenario as in Fig. 7(a). Such a result is because, in Fig. 7(a), the users care more about the video delay than the users in Fig. 6, i.e., g_D for the users in Fig. 7(a) is higher. It suggests that, when the users are more concerned about the streaming delay, providing better network quality brings the service provider more gain than providing better video quality. In Fig. 7(b), all six mobile users' gain weighting factor for streaming delay $g_D = 0.1$, but two of them have a quality gain factor $g_Q = 0.1$, two of them have a quality gain factor $g_Q = 0.15$, and the rest two of them are with a

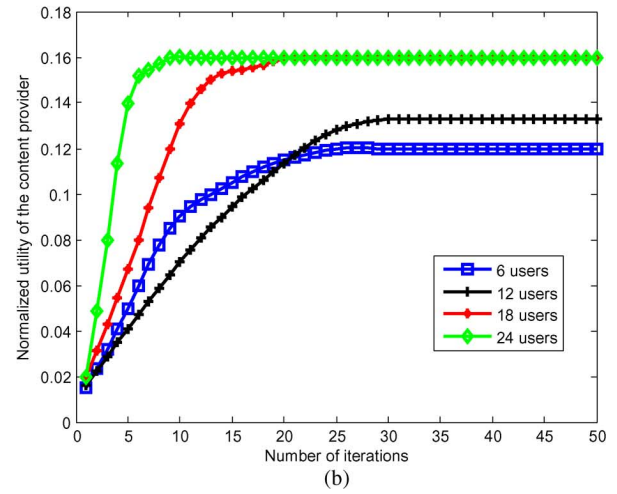
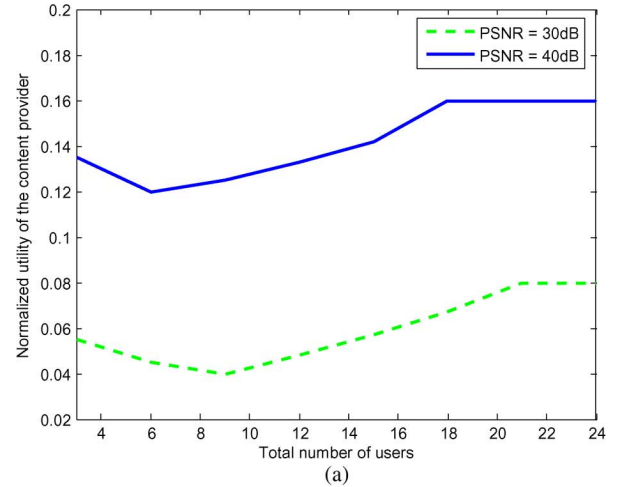


Fig. 8. Convergence speed and content provider's utility versus number of the mobile users. (a) Utility versus number of users. (b) Convergence speed for PSNR = 40 dB.

delay factor $g_Q = 0.2/\text{ms}$. Comparing Fig. 7(a) with (b), it is obvious that, when some of the mobile users care more about the video quality, providing competitive video quality gives lot more gain to the service provider. Comparing Fig. 7(c) and (d) with Fig. 7(a) and (b), we can also see that, when the users care more about the video quality (higher g_D and g_Q), the content owner's utility will be higher since the content owner can provide better PSNR and less delay than the redistribution network can serve. However, when g_D is large, the service provider's utility will significantly degrade when the network quality is bad.

Fig. 8 shows how the total number of mobile users within the same range influences the service provider's maximal utility. To clearly show the factor of user density, the utilities shown in Fig. 8 are normalized to that of the six users, i.e., the service provider's utility when there are 12 mobile users is divided by half. We fix $M - K = 25$, $g_Q = 0.1$, and $g_D = 0.1/\text{ms}$, and the mobile users are randomly located in a 100 m by 100 m square. In Fig. 8(a), we can see that, when the number of users is large enough (six users for PSNR = 40 dB; nine users for PSNR = 30 dB), the utility of the service provider is increasing with respect to the number of users. This is because the bandwidth dedicated for redistribution is fixed, and when

there are more users in the network, the bandwidth for each subscriber is smaller. Hence, the transmission rate of each subscriber is getting smaller and results in poorer video quality when the total number of users is increasing. However, when there are only a few users, (three users for PSNR = 40 dB; six users for PSNR = 30 dB), the utility of the service provider is slightly decreasing when the number of users is increasing. This is because, when the total number of users is small, there is not enough competition among the subscribers; therefore, the data service price of the redistribution network is not very low compared with that of the service provider. Hence, the users will tend to directly subscribe from the content owner. Fig. 8(b) shows the convergence speed versus the number of users in the network when the video PSNR = 40 dB. We can see that, when there are a lot of users (more than 18 users), the convergence speed is fast because the users quickly realize that the bandwidth of the redistribution channel is not enough for streaming video to many secondary buyers, and when there are less number of users, a larger number of users result in a more complicated behavior in the redistribution network; hence, the convergence speed is slower.

VI. CONCLUSION

In this paper, we have investigated the optimal pricing for mobile video data by analyzing the video redistribution network between data-plan subscribers and nonsubscribers. We have first analyzed the equilibrium price of the video stream redistributed by the subscribers given the number of subscribers and secondary buyers. Consequently, the results provide a guideline for the content owner to prevent the redistribution behavior and to maximize the service provider's payoff. The redistribution behavior has been modeled as a Stackelburg game, and we have analyzed the optimal strategies of both subscribers and secondary buyers. From the simulation results, a secondary buyer will tend to buy more power from subscribers with better channel to maximize his/her utility. If the total number of the subscribers increases, a secondary buyer can obtain a larger utility value, and the payment to each subscriber is reduced due to a more severe competition among the subscribers. Also, when the mobile phone network is crowded, a secondary buyer tends to purchase the video stream from fewer subscribers, and the price for the streaming service can be higher. Nevertheless, the service provider should always offer high-quality video stream to prevent the illegal redistribution of video via such redistribution networks.

Next, we have extended the model by including the content owner in the game and letting the mobile phone users decide whether to subscribe to the data plan. In the extended model, we model the dynamics between the content owner and the users who are interested in the video content, and study how the content owner (the service provider) sets the price for the data plan to maximize his/her overall income. We have used the evolutionary game theory to analyze the evolution of the mobile users' behavior and have derived the evolutionarily stable equilibrium, which leads to the optimal price for the content owner to maximize his/her total income.

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