Throughput and Delay Analysis for Convergecast with MIMO in Wireless Networks

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Abstract—This paper investigates throughput and delay based on a traffic pattern, called converge-cast, where each of the n nodes in the network acts as a destination with k randomly chosen sources corresponding to it. Adopting Multiple-Input-Multiple-Output (MIMO) technology, we devise two many-to-one cooperative schemes under converge-cast for both *static* and *mobile* ad hoc networks (MANETs), respectively. We call them Convergimo Schemes. In static networks, our Convergimo scheme highly utilizes hierarchical cooperation MIMO transmission. This feature overcomes the bottleneck which hinders converge-cast traffic from yielding ideal performance in traditional ad hoc network, by turning the originally interfering signals into interference-resistant ones. It helps to achieve an aggregate throughput up to $\Omega(n^{1-\epsilon})$ for any $\epsilon > 0$. In the mobile ad hoc case, our Convergimo scheme characterizes on *joint* transmission from *multiple* nodes to *multiple* receivers. With optimal network division where the number of nodes per cell is constantly bounded, the achievable per-node throughput can reach $\Theta(1)$ with the corresponding delay reduced to $\Theta(k)$. The gain comes from the strong and intelligent cooperation between nodes in our scheme, along with the maximum number of concurrent active cells and the shortest waiting time before transmission for each node within a cell. This increases the chances for each destination to receive the data it needs with minimum overhead on extra transmission. Moreover, our converge-based analysis well unifies and generalizes previous work since the results derived from converge-cast in our schemes can also cover other traffic patterns. Last but not least, our schemes are of interest not only from a theoretical perspective but also provide useful theoretical guidelines to future design of MIMO schemes in wireless networks.

Index Terms—Convergecast, Throughput, Delay, MIMO.

1 INTRODUCTION

Since the seminal work of Kumar [1] *et al.*, who showed that the optimal static unicast capacity is $\Theta(\frac{1}{\sqrt{n}})$ and $\Theta(\frac{1}{\sqrt{n\log n}})$for random network, capacity analysis of ad hoc networks have triggered great interest. Later on, Grossglauser and Tse [2] demonstrated that $\Theta(1)$ capacity per source-destination (S-D) pair is achievable if taking mobility of the network into account, but the packet has to endure a delay going to infinity. Due to the phenomenon that larger capacity is at the cost of a larger delay, some analysis on capacity-delay tradeoffs arises. One interesting work is from Neely and Modiano [3] who introduced redundant packets transmission through multiple opportunistic paths to reduce delay while a decrease on capacity is also incurred. Under *i.i.d.* mobility, the per-node capacity is shown to be $T(n) = \Theta(1)$ and delay $D(n)$ yielded to scale as $\Theta(n \cdot T(n))$ [3]. Later work also studied the tradeoff between capacity and delay, where nodes either perform traditional operations such as storage, replication and forwarding ([4]- [6]) or transmit through coding or infrastructure support ([7]- [9]).

However, all the results above strongly rely on the assumption that all the concurrent transmissions are always interfering with others. This becomes a limitation which largely constrains the capacity. In contrast, MIMO enables nodes to perform cooperative communication by turning mutually interfering signals into useful ones, where the gain of capacity can then be obtained. The gain was well demonstrated by Aeron *et al.* [10] who presented a MIMO collaborative strategy which achieves a per-node capacity of $\Theta(n^{-1/3})$. Following that, Özgür *et al.* [16] constructed a hierarchical cooperative scheme relying on distributed MIMO communications to achieve a linear capacity scaling. It turned out that nearly all the interferences can be canceled through hierarchical cooperation. Thereon, multicast scaling was taken into account in [15] under hierarchical cooperation which achieves an aggregate capacity of $\Omega\left(\left(\frac{n}{k}\right)^{1-\epsilon}\right)$ for any $\epsilon > 0$. This also achieved a gain on capacity compared with previous works on multicast such as [11]- [14].

While the tradeoff for *unicast and multicast* traffic pattern have been extensively studied in previous work, *converge-cast* is still a relatively new concept and under active research. *Converge-cast* refers to a communication pattern in which the flow of data from a set of nodes transmit to a single node, either directly or over multi-hop routes. Recently, there appeared many new applications such as real-time multimedia, battlefield com-

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munications and rescue operations that impose stringent capacity-delay requirements on converge-cast.

In this paper, we jointly consider the effect of converge-cast and cooperative strategies on asymptotic performance of networks. The motivations come from the following reasons: **1.** Although there have been some researches on converge-cast (such as [17], [18], [19], [20]), their major concern is limited to the extreme case where all nodes flow data to a single sink in the network. However, a wide range of applications such as machine failure diagnosis, pollutant detection and supply chain management may require multiple such converge-cast groups existing in parallel in the network rather than a single one. **2.** Vast space of further improvement on performance can be discovered in converge-cast, due to its convergent process. **3.** Since distinctive sources may transmit different data to their common destination, such traffic pattern can be treated as a generalized reversed "multicast". To our best knowledge, there are no previous study on the network performance under converge-cast with MIMO.

Concentrating on throughput and delay performance in this paper, we propose a new type of many-to-one cooperative schemes with MIMO in both static and mobile networks, from the perspective of converge-cast. We call them Convergimo schemes. For Convergimo scheme in a static network, the whole network is divided into clusters with equal number of nodes in each of them. Communications between clusters are conducted through distributed MIMO transmissions combined with multi-hop strategy while within a cluster it is operated through joint transmission of multiple nodes to others. Through hierarchical operation, each cluster can be treated as a subnetwork and further divided into smaller clusters. In a traditional ad hoc network, only one transmission can be active at a time while all the adjacent transmissions are treated as interference. This imposes a significant bottleneck on converge-cast and makes it impossible to achieve ideal performance. However, this bottleneck can be removed with the adoption of MIMO. by transforming interfering signals into useful ones to the receivers, during hierarchical cooperative transmissions.

Under MANETs where hierarchical cooperation cannot be established due to the mobility of nodes, we devise another Convergimo scheme where the network is still divided into equal cells. In each time slot, multiple nodes that possess information for the same destination are allowed for joint transmission to other nodes within the cell. Other nodes will receive a combination of the information from these transmitters due to the effect of MIMO through fading channels. This procedure continues, with the number of nodes that hold such mixed information increases, untill all the destinations receive sufficient mixed information that can be decoded with high probability.

Our main contributions can be summarized as follows:

- Our Convergimo scheme in a static network breaks the bottleneck hindering converge-cast from achieving ideal performance in a traditional network by converting adjacent interfering signals into useful ones. The achievable aggregate throughput can be up to $\Omega(n^{1-\epsilon})$ for any $\epsilon > 0$, which nearly approaches the upper-bound.
- For our Convergimo scheme under MANETs, with optimal network division, the per-node throughput is $\Theta(1)$ with the corresponding delay reduced to $\Theta(k)$.
- Our results well unify and generalize some previous works since all of them can be easily applied to other traffic modes. Especially, our scheme in MANETs breaks the vacancy of such MIMO scheme design remaining in mobile networks before.

The rest of the paper is organized as follows. In Section 2, we present the models and definitions. In Section 3 and Section 4, we describe our Convergimo schemes under static and mobile ad hoc networks, respectively. The corresponding throughput and delay achieved based on the two schemes are also presented in detail in these two sections. All the results are further discussed in Section 5.Finally, we present concluding remarks in section 6.

2 Models and Definitions

2.1 Network Model

In this paper, we consider an ad hoc network where nodes are randomly positioned in a unit square.

Traffic Pattern: In converge-cast scenario, we assume n nodes located in the network with each one serving as a destination. For each destination node, there are k *randomly and independently* chosen sources. Since the total number of nodes is n, there must be some sources shared among different converge-cast sessions. For each destination, it will receive distinctive packets from its k sources. In multicast, all the packets sent out from a source node are the same while in convergecast, the packets from those k sources may be totally different and all of them are indispensable to form the complete information. Moreover, the data rates of each edge of the spanning tree in multicast are all same while they are different in each edge in convergecast.

Physical Layer model: We assume that communication takes place over a channel with limited bandwidth W. Each node has a power budget P, which is assumed to be a constant[1]. The channel gain between two nodes v_i and v_j at time t is given by:

$$h_{ij}[t] = \sqrt{G} d_{ij}^{-\alpha/2} e^{j\theta_{ij}[t]}, \tag{1}$$

where d_{ij} is the distance between the nodes, $\theta_{ij}[t]$ is the random phase at time t, uniformly distributed in

[1] The power budget indicates the transmitting limit in our model and the actual power consumption can be expressed as $Pf(\cdot)$, where $f(\cdot)$ is a function related to the parameters such as number of nodes, number of sources for each converge-cast session and etc. When we focus on the result of power consumption in order sense, we can simply remove the term P from the expression.

$[0, 2\pi)$. $\{\theta_{ik}[t]\}$ are i.i.d. random processes across all i and k, independent of each other. G and the path loss propagation $\alpha \geq 2$ are assumed to be constants. Then, the signal received by node i at time t can be expressed as

$$Y_i[t] = \sum_{j \in \mathbb{T}[t]} h_{ij}[t] X_j[t] + Z_i[t] + I_i[t], \tag{2}$$

where $Y_i[t]$ is the signal received by node v_i at time t, $\mathbb{T}[t]$ represents the set of active senders transmitting signals to v_i, which can be added constructively, $Z_i[t]$ is the additive white Gaussian noise at v_i with variance N_0 per symbol and $I_i[t]$ is the interference from the nodes.

Since stochastic analysis and optimization are not the main focus in this paper, in the following discussions, we will simplify the notation by suppressing the dependency of the channel gains on the time index t.

MIMO Technology: We adopt multiple-input and multiple-output, or MIMO Technology in this paper. In radio, MIMO represents the use of multiple antennas at both the transmitter and receiver to improve communication performance. It is one of several forms of smart antenna technology. MIMO technology has attracted attention in wireless communications, because it offers significant increases in data throughput and link range without additional bandwidth or transmit power.

Number of Antennas: Moreover, we assume each node is equipped with one antenna. We do not consider the case where each node has multiple antennas for the following two reasons: 1. If each node is assumed to have constant bounded number of antennas, say, c antennas, then the throughput is c times that achieved in single-antenna case, which does not change the throughput order; 2. If each node has n_r antennas where n_r scales with n, then the throughput achieved in order sense is n_r times that of single-antenna case. This is trivial and assuming n_r antennas on one node is not realistic.

2.2 Definitions

Converge-cast Session: a converge-cast session is defined as the set composed of one destination and its corresponding k sources.

Delay: Delay is defined as the time a destination takes to receive all the packets from its corresponding k sources. The averaging is over all bits (or packets) transmitted in the network.

Throughput: Denoting $m(t)$ as the number of packets from sources that a destination receives in t time slots. Then, the long term per-node throughput, denoted by λ, is defined as

$$\lambda = \liminf_{t \to \infty} \frac{m(t)}{t}.$$

And the aggregate throughput is $\Lambda = n\lambda$.

2.3 Notations

In table 1, we list all the parameters that will be used in later analysis, proofs and discussions.

TABLE 1: Notations

Notation	Definition
n	The total number of nodes in the network.
k	The number of sources for each destination in the network.
h	The number of layers a network is divided into.
i	The ith layer of the network, where $1 \leq i \leq h$.
n_i	The number of nodes in the ith layer.
k_i	The number of sources for each destination node in the ith layer.
n_{c_i}	The number of clusters in the ith layer.
k_{c_i}	The number of source clusters in the ith layer.
t_i	The number of converge-cast sessions in the ith layer.
T_i	The aggregate throughput at layer i in static network.
$D(i,k)$	The average delay to complete a converge-cast session for a destination at layer i.
B_i	The minimum amount of data a node needs to send at layer i. We can also call it bulk size.
M	The average number of nodes in each cell.

3 Convergimo Scheme under Static Networks

In this section, we will design a cooperative scheme with MIMO under static networks. Then we will analyze the throughput and delay achieved under the scheme.

3.1 Convergimo Scheme 1 under Static Networks

As is shown in [16], hierarchical cooperation can achieve better throughput scaling than classical multi-hop schemes under certain assumptions on the channel model in static wireless network. This motivates us to design a hierarchical scheme which can be applied to converge-cast.

3.1.1 Scheduling Algorithm

Under hierarchical schemes, a network is divided into clusters with equal number of nodes in each one. Each cluster is then treated as a *subnetwork* and we can further divide the subnetwork into smaller clusters. Take layer i for example. There are totally n_i nodes at this layer. Treating it as a whole network, it is divided into n_{c_i} clusters with n_i/n_{c_i} nodes located in each of them. At layer $i-1$, each of those n_{c_i} clusters is further regarded as a whole network with totally number of nodes of n_i/n_{c_i}. The network is further divided into $n_{c_{i-1}}$ clusters. With recursion operations, the procedure goes on until the network is divided into h layers with the original network at the hth layer and the 1st layer at the bottom one. A scheduling algorithm can be designed on each subnetwork at each layer. The algorithm keeps executing from layer to layer, the process of which is similar per layer per cluster but with a larger scale as the number of layer i increase from 1 to h. The procedure continues until all the layers have finished the algorithm. A whole view of the hierarchical structure of Convergimo Scheme 1 is shown in 1.1 in **supplementary file**.

Since the algorithm is similar at each layer but with different scale, we will present our recursive cooperative scheme 1 at a particular layer i. For a specific layer i, the number of sources k_i for a destination node can be expressed as $k_i = k / \prod_{k=i+1}^{k=h-1} n_{c_k}$. Note that these k_i sources are part of some k source nodes in the original network. Moreover, at layer i, the scheme is divided into three steps described as follows.

Step 1. Preparing for Cooperation with Recursion: Since there are k_i source nodes belonging to one session at layer i, under converge-cast, they must distribute their packets[2] to other nodes in the same cluster. For each node in the cluster, the k_{i-1} sources jointly transmit their packets to other nodes in the cluster, which receives a linear combination of that bit mixed with channel coefficients. The process keeps until all the other nodes except for these k_{i-1} sources receive the packets from them. Note that as for each transmission from the k_{i-1} sources to a specific node, the process is a many-to-one transmission and this is equivalent to dividing the current cluster into smaller-size clusters and the similar procedure executes in a smaller cluster. Note that our algorithm starts from the bottom layer, *i.e.*, layer 1 of the network and continues to a higher layer until layer h.

Step 2. Multi-hop MIMO Transmissions: We construct a convergecast tree (CT) spanning from source nodes to its associated their destination node. Several source clusters start a series of MIMO transmissions to reach their common destination clusters in multi-hop manner. Since each source cluster has $\frac{n_{i-1}k_{i-1}}{k_i}$ packets to send in one time slot, due to MIMO, several source clusters are allowed for concurrent transmission to one cluster at the same time slot. To achieve asymptotically optimal converge-cast capacity, we conduct the three substeps presented below, spanning from source clusters S_{ij}s to their common destination clusters D_i. Here $1 \leq j \leq k_{c_i}$. Denote $P_i = \{S_{ij}, D_i, 1 \leq j \leq k_{c_i}\}$.

1) Constructing the Euclidean spanning tree $\mathscr{E}_{EST}$: Firstly we divide the unit square into multi-level cells[3] with side length $\frac{1}{2^g}$, where $g = t - 1$, $t = \lceil \log_4 k \rceil$. For each cell that contains $s \geq 2$ clusters in P_i, we randomly select a cluster p_{ij}. For any other $p_{ik} (k \neq j)$ in the cell, let $\mathscr{E}_{EST} \rightarrow \mathscr{E}_{EST} \cup \{\overline{p_{ix}p_{iy}}\}$ and $\mathcal{P} \rightarrow \mathcal{P} - \{p_{ik}\}$. Here $\overline{p_{ix}p_{iy}}$ represents an edge connect by the two cluster p_{ix} and p_{iy}. Subsequently we conduct this process by letting $g = t-2, \ldots, 1, 0$.
2) Getting the Manhattan routing tree $\mathscr{E}_{MRT}$: for each edge $\overline{uv}$ in $\mathscr{E}_{EST}$, assume that the coordinates of u and v are (i_u, j_u) and (i_v, j_v), respectively. We then find a cluster w whose coordinate is (i_u, j_v). Afterwards, $\mathscr{E}_{MRT} \rightarrow \mathscr{E}_{MRT} \cup \{\overline{uw}\}$, $\mathscr{E}_{MRT} \rightarrow \mathscr{E}_{MRT} \cup \{\overline{wv}\}$, $\mathscr{E}_{MRT} \rightarrow \mathscr{E}_{MRT} - \{\overline{uv}\}$.
3) Obtaining the the converge-cast tree for P_i, denoted as CT(P_i), for each edge $\overline{uw}$ in $\mathscr{E}_{MRT}$, we connect clusters crossed by $\overline{uw}$ in sequence to form a path, denoted as $E(u, w)$. Then $\mathscr{E}_{CT} \rightarrow \mathscr{E}_{CT} \cup E(u, w)$, $\mathscr{E}_{MRT} \rightarrow \mathscr{E}_{MRT} \cup \{\overline{uw}\}$. Finally, $\mathscr{E}_{CT}$ is the set of edges of CT(P_i).

Note that the structure of both Euclidean spanning tree and Manhattan routing tree are invented from [10], based on a good approximation of a minimum connected dominating set (MCDS) of a random network. However, the direction and the amount of data flow in CT is different from those in [10]. Consider at layer i, there are k_i sources distributed in k_{c_i} source clusters. For any one of the k_{c_i} source clusters, denoted by j, we assume there are k_{i_j} sources located in it. Obviously, $\sum_{j=1}^{k_{c_i}} = k_i$. For a source cluster j that has k_{i_j} sources, k_{i_j}/k_i nodes in this cluster participate in joint transmission of the packets to other clusters. Moreover, for a cluster denoted by c_3, which will receive the data from its two adjacent clusters, denoted by c_1 and c_2, if a/k_i of c_1 nodes are allowed for joint transmission in c_1 and b/k_i of c_2 nodes are allowed for joint transmission in c_2, then $(a+b)/k_i$ of the nodes in c_3 will be active for joint transmission to next cluster. Under this transmission rule, when the data is finally flowed to the destination cluster, all the nodes in that cluster will be active for joint receiving. In supplementary file, Figure 2 shows a simple example of the data flow on such converge-cast tree (CT).

Step 3. Cooperative Reception: Given the total number of converge-cast sessions t_i at layer i, consider a particular node in the cluster. It can receive $\frac{t_i k_i}{n_i}$ packets from other nodes, with each of them contributing $\frac{t_i k_i}{n_i n_{i-1}}$ packets. Considering n_{i-1} destinations in each cluster, the traffic load are $\frac{t_i k_i n_{i-1}}{n_i}$ packets. Since the data exchanges only involve intra-cluster communication, they can work according to 9-TDMA scheme where the cells which are located 3 cells away from each other can be active concurrently.

3.1.2 *Throughput and Delay Analysis under Convergimo Scheme 1*

Now we focus on throughput and delay that can be achieved under the scheme presented in 3.1.1. We first derive the upper-bound of throughput and our main results as follows.

Lemma 1: Under converge-cast, with each of the n nodes in the network acting as destination and receiving packets from its distinctive k sources, the aggregate capacity is upper-bounded by

$$\sum_{i=1}^{n} \lambda_i \leq C n \log n, \tag{3}$$

where $C > 0$ is a constant independent of n.

Proof: Provided in 1.4 in **supplementary file**. □

[2] Note that the size of packet in our cooperative scheme 1 differs at different layers. Each node divides a packet into $\frac{n_{i-1}}{k_i}$ packets. Each of these packets is then used in the next layer $i-1$. In other words, the size of a packet at layer i is n_{i-1}/k_i times larger than that at layer $i-1$ and the packets evolve across layers with their sizes changing in this way.

[3] It is already proved in [11] that through multi-level cells, the Euclidean length of the constructed tree is of same order of the Euclidean length of Euclidean minimum spanning tree.

Theorem 1: In static wireless networks, by adopting our Convergimo scheme 1, we can achieve an aggregate throughput of

$$\Lambda = \widetilde{\Theta}\left(n^{\frac{2h-2}{2h-1}} \cdot k^{-\frac{1}{2h-1}}\right) \tag{4}$$

with the delay of

$$\mathbb{E}[T] = \begin{cases} \widetilde{\Theta}\left(n^{\frac{2h^2-4h+3}{2h-1}} \cdot k^{-\frac{2h^2-2h-1}{2h-1}}\right), & \text{if } k = \Omega(n^{\frac{1}{2h-2}}) \\ \widetilde{\Theta}\left(n^{\frac{h^2-2h+2}{2h-1}} k^{\frac{h^2-4h+3}{2h-1}}\right), & \text{if } k = O(n^{\frac{1}{2h-2}}) \end{cases} \tag{5}$$

To prove Theorem 1, we will first introduce the following lemmas.

Lemma 2: (Lemma 4.3 in [15]) By 9-TDMA scheme, when $\alpha > 2$, one node in each cluster has a chance to operate data exchanges at a constant transmission rate. Also when $\alpha > 2$, the interfering power received by a node from the simultaneously operating clusters is upper-bounded by a constant.

Lemma 3: Given k_i independently and uniformly distributed source nodes in the network at layer i, the number of source clusters k_{c_i} is given by

$$k_{c_i} = \begin{cases} \Theta\left(k_i\right), & \text{if } k_i = O(n_{c_i}) \\ \Theta\left(\frac{n_i}{n_{i-1}}\right), & \text{if } k_i = \Omega(n_{c_i}) \end{cases}. \tag{6}$$

Proof: The proof is similar to that of Lemma 4.5 in [15] and we do not present the proof here. □

Lemma 4: When $t_i k_i = O\left((n_{c_i})^{p_2}\right)$ holds for all layer i, where $2 \le i \le h$ and p_2 is a positive constant,

- if $k_i = \Omega(n_i \log n_{c_i})$, then $k_{i-1} = \Theta\left(\frac{k_i}{n_{c_i}}\right)$ w.h.p.
- if $k_i = O(n_i \log n_{c_i})$, then $k_{i-1} = O\left(\frac{k_i}{n_{c_i}}\right)$ w.h.p.

Proof: The proof is similar to that of Lemma 4.6 in [15] and we do not present the detailed proof here. □

Consider the three steps in our scheme at layer i. Assume an aggregate converge-cast throughput $\widetilde{\Theta}\left(n_{i-1}^{a} k_{i-1}^{b}\right)$ is achievable at layer $i-1$ *w.h.p.*, where $0 \le a \le 1$, $-1 \le b \le 0$ and $a+b<0$. It is easy to obtain that the total time to complete $k_i t_i$ traffic loads is

$$\widetilde{\Theta}\left(\frac{k_i t_i n_{i-1}^{1-a}}{n_i k_{i-1}^{b}}\right) + O\left(k_i t_i \sqrt{\frac{k_i}{k_{i-1} n_i n_{i-1}}}\right) + \Theta\left(\frac{k_i t_i n_{i-1}^{1-a}}{n_i}\right). \tag{7}$$

Hence, the throughput can be expressed as

$$\begin{aligned} T_i &= \frac{k_i t_i}{\widetilde{\Theta}\left(\frac{k_i t_i n_{i-1}^{1-a}}{n_i k_{i-1}^{b}}\right) + O\left(k_i t_i \sqrt{\frac{k_i}{k_{i-1} n_i n_{i-1}}}\right) + \Theta\left(\frac{k_i t_i n_{i-1}^{1-a}}{n_i}\right)} \\ &= \widetilde{\Theta}\left(\frac{n_i n_{i-1}}{\sqrt{n_i n_{i-1} k_{c_i}} + n_{i-1}^{2-a} k_{i-1}^{-b}}\right). \end{aligned} \tag{8}$$

Due to page limitations, we do not present the detailed proof here. Instead, we will show in 1.5 in **supplementary file** how to obtain the optimal throughput at the network at layer h, given T_i in the above equation.

4 Convergimo Scheme under MANETs

Due to the mobility characteristics of nodes, the network performance may be quite different from that in static ones. In the following subsections, we will introduce the mobility model and present another scheme that is suitable for mobile networks. Then, we will give our analysis on throughput and delay obtained from the scheme.

4.1 Mobility Model

We introduce *two-dimensional i.i.d. mobility model* into the network, i.e., n nodes are uniformly distributed in the network. At the beginning of each time slot, each node randomly chooses a point in the unit square and moves there. In this model, we assume that the nodes move quickly so that the nodes' positions are independent from time slot to time slot. We also define it as *fast mobility model* where the mobility of nodes is at the same time-scale as that of data transmission.

Remark 1: Although the *i.i.d.* mobility model may appear to be unrealistic, it has been widely adopted in the literature because of its mathematical tractability. Due to the its property that the position of each node at different time slots and between different nodes are both independent, it will simplify the analysis to some extent. Note that other mobility models, such as the Brownian motion, random walk and random waypoint, possess the Markov property between time slots, which complicates the analysis.

4.2 Convergimo Scheme 2 under MANETs

It is impossible to construct a hierarchical scheme under mobile networks. Since the relationship determined in the current time slot between nodes may be destroyed in the next one due to the randomness incurred by mobility. Hence, we need to design a new scheme that can take advantage of mobility of the nodes.

4.2.1 Convergimo Scheme 2

We divide the whole network into c cells such that there are M nodes in each cell on average. To avoid the interference incurred to the network from the neighboring cells, we adopt the 9-TDMA strategy illustrated in Section 3 again.

- Each cell becomes active once every c_0 time slots. In an active cell, transmission occurs among the nodes within the same cell.
- In an active cell, in each time slot, if there exist both a destination and some of its sources, then we call there are *sources-destination pair* in the cell. If there are several such pairs in the cell, then we randomly choose one pair, and let all these sources in this pair form an antenna array and jointly send their packets to their common destination as well as all the other nodes in that cell.

- If there are no sources-destination pairs in the cell, choose the maximum number of sources that belong to the same destination in that cell. Then, the chosen sources jointly send their packets to all the other nodes in the same cell.
- If there are neither sources-destination pairs nor sources that belong to the same destination in the cell, then choose the maximum number of relays which hold the packets that are to be transmitted to the same destination. Those chosen relays then jointly send their packets to all the other nodes in the same cell.

A simple illustration of our scheme is shown in Figure 3 in **supplementary file**.

4.3 Analysis of Throughput and Delay under Convergimo Scheme 2

In this subsection, we will analyze the achievable throughput and delay under our proposed scheme 2. First, we will first compute the bound of achievable delay and then analyze the corresponding throughput.

The main results obtained under scheme 2 is presented in the following theorem.

Theorem 2: Suppose $k = o(n)$, then under Convergimo scheme 2, with the optimal network division $M = \Theta(1)$, we can achieve ideal performance on both the average delay required for a destination to receive packets from all its k corresponding sources and the per-node throughput, listed as follows:

$$\begin{cases} \lambda = \Theta\left(\frac{1}{\log(n)}\right), \mathbb{E}[D_N] = \Theta(\log(n)) & \text{if } k = o(\log n) \\ \lambda = \Theta\left(1\right), \mathbb{E}[D_N] = \Theta(k) & \text{if } k = \omega(\log n) \end{cases}. \tag{9}$$

To prove Theorem 2, we turn to the proof for delay in 4.3.1 first and then prove the throughput in 4.3.2.

4.3.1 Analysis on Delay

Before the proof of delay, we fist introduce the following two lemmas.

Lemma 5: Consider n nodes uniformly distributed in the network area. The network is divided into c identical cells. Then, the number of nodes in each cell is $M = \Theta\left(\frac{n}{c}\right)$ w.h.p. if $\lim\limits_{n\to\infty} \frac{n}{c\log c} = \infty$.

Proof: Provided in 1.6 in **supplementary file**. □

Lemma 6: As for a destination node, the condition that it can successfully decode the packets from all its k sources is that there should be at least k different linear combinations of these packets in its receiving buffer and the coefficient vectors of these k combinations are linearly independent of each other[4].

Viewing from the perspective of network coding, the central problem arises: *how long does it take for a destination node to receive at least $\Theta(k)$ combinationon average?* If denoting the whole time as D_N, then $\mathbb{E}[D_N] \leq \mathbb{E}[D_1] + \mathbb{E}[D_2]$, where $\mathbb{E}[D_1]$ and $\mathbb{E}[D_2]$ represent the time required for all nodes in the network to have one "packet" of the sources belong to that destination and the time required for the destination to receive $\Theta(k)$ packets given that all the other nodes already hold an "packet", respectively.

Lemma 7: The average delay for letting all nodes in the network to have one "packet" of the sources belonging to the same destination is bounded by

$$\mathbb{E}[D_1] = \begin{cases} \Theta(M) & \text{if } M = \omega(\log(n)) \\ \Theta(\log n) & \text{if } M = o(\log(n)) \end{cases}. \tag{10}$$

Proof: Provided in 1.7 in **supplementary file**. □

As for $\mathbb{E}\{D_2\}$, it is easy to know that it takes a single destination k slots to receive k distinctive "encoded" packets given that all the nodes in the network already hold one of them. And consider the fact that each destination in one cell will have such chance once every M time slots, we have $\mathbb{E}\{D_2\} = \Theta(Mk)$. Therefore, the total delay achieved under our scheme is

$$\mathbb{E}[D_N] \leq \mathbb{E}[D_1]+\mathbb{E}[D_2] = \begin{cases} \Theta(Mk) & \text{if } k = \omega(\log(n)) \\ \Theta(\log(n)) & \text{if } k = o(\log(n)) \end{cases}. \tag{11}$$

4.3.2 Analysis on Throughput

Lemma 8: Under Convergimo scheme 2, we can achieve a per-node converge-cast throughput of

$$\lambda = \begin{cases} \Theta\left(\frac{1}{\log(n)}\right) & \text{if } k = o(\log(n)) \\ \Theta\left(\frac{1}{M}\right) & \text{if } k = \omega(\log(n)) \end{cases}. \tag{12}$$

in an MANET.

Proof: Provided in 1.10 in **supplementary file**. □

Notice that both the throughput and delay are optimized when $M = \Theta(1)$, which renders the results presented in Theorem 2.

5 Discussion

5.1 The advantage of Our Convergimo Schemes

In **static** network, our Convergimo scheme allows for concurrent transmission, which converts the interfering signals into useful ones. This reduces the interference level to an extensive degree and therefore undoubtedly leads to an improvement on throughput. More specifically, we have shown in our analysis that to achieve the optimal throughput, the network division should be $n_{i-1} = \widetilde{\Theta}\left(k_i^{\frac{-b}{a+b-2}} n_i^{\frac{b-1}{a+b-2}}\right)$ if $n_{c_i} = O(k_i)$ and $n_{i-1} = \widetilde{\Theta}\left((n_i k_i)^{\frac{1}{3-2a}}\right)$ if $n_{c_i} = \Omega(k_i)$. However, this optimal result is achieved under a given h. Varying h will lead to different optimal network division. Our aggregate

[4] In mobile networks, to obtain the channel status information (CSI) of other nodes, a training sequence is contained in each packet. A destination can then recover CSI through these training sequence contained in each packets. We assume it is of the equal size of a packet and the packet size is sufficiently small compared to the total number of nodes in the network. Thus, the mobility of nodes during the acquirement of a training sequence can be neglected, if compared to data transmission time.

Network	Traffic	Throughput	Delay
Static Network	Unicast	Our Convergimo Scheme 1: $\widetilde{\Theta}\left(n^{\frac{-1}{2h-1}}\right)$	$\widetilde{\Theta}\left(n^{\frac{h^2-4h+3}{2h-1}}\right)$
		Hierarchical Cooperation Scheme in [16]: $\Theta\left(\frac{n^{-b}}{\log n}\right)$ $(0 < b < 1)$	$\Theta\left(n^b \log n\right)$ $(0 < b < 1)$
	Converge-cast	Our Convergimo Scheme 1: $\Theta(1)$	$\Theta(1)$
		Antenna Sharing Approach in[17]: $\Theta(\log n)$	-
		Collision-Free Strategies in [18]: $\Theta\left(\frac{1}{\log n}\right)$	-
		Cooperative Time-Reversal Communication Strategy in [19]: $\Theta(\frac{\log n}{n})$ and $\Theta(1)$	-
		Collection/Aggregation Method in [20]: $\Theta\left(\frac{1}{\log n}\right)$	-
Mobile Network	Unicast	Our Convergimo Scheme 2: $\Theta\left(\frac{1}{\log(n)}\right)$	$\Theta(\log(n))$
		Random Linear Coding Scheme in [7]: $\Theta\left(\frac{1}{n}\right)$	$\Theta(\log n)$
	Multicast	Our Convergimo Scheme 2: $\Theta\left(\frac{1}{n\log(n)}\right)$	$\Theta(\log(n))$
		Random Linear Coding Scheme in [7]: $\Theta\left(\frac{1}{n}\right)$	$\Theta(\log n)$
	Broadcast	Our Convergimo Scheme 2: $\Theta\left(\frac{1}{n\log(n)}\right)$	$\Theta(\log(n))$
		Random Linear Coding Scheme in [7]: $\Theta\left(\frac{1}{n}\right)$	$\Theta(\log(n))$

TABLE 2: Throughput and Delay of converge-cast and that extended to unicast, multicast and broadcast under our schemes in both static and mobile networks. Comparison is also made between our results and previous ones.

throughput result is obtained based on this optimal network partition at each layer with recursion operations from layer 1 to layer h. Since our major concern is the throughput, the optimal h should be the one that maximizes the throughput. Given the number of hierarchical layers, the aggregate throughput in our paper is $\widetilde{\Theta}\left(n^{\frac{2h-2}{2h-1}} \cdot k^{-\frac{1}{2h-1}}\right)$. It can be seen from this equation that the throughput increases as h becomes large. When the number of layers h are sufficiently large in Convergimo scheme 1, the aggregate throughput can reach $\Theta(n)$. This is close to the upper-bound with difference of only a $\log(n)$ factor. In **MANETs**, with further observation on our scheme 2, we can find it is to some extent equivalent to a "flooding" algorithm but with more intelligent transmission. However, in previous flooding algorithm, packets are simply broadcasted arbitrarily to other nodes in the cell, regardless of whether the receivers are destinations of those packets. This undoubtedly leads to some unnecessary waste on the number of transmission, which incurs sacrifice on throughput.

5.2 Delay-throughput Tradeoff

Static network: By THEOREM 1, we obtain the delay/throughput tradeoff, as is shown as follows:

$$\begin{cases} \widetilde{\Theta}\left(n^{\frac{2h^2-4h+4}{2h-1}} \cdot k^{-\frac{2h^2-2h}{2h-1}}\right), & \text{if } k = \Omega(n^{\frac{1}{2h-2}}) \\ \widetilde{\Theta}\left(n^{\frac{h^2-2h+1}{2h-1}} k^{\frac{h^2-4h+2}{2h-1}}\right), & \text{if } k = O(n^{\frac{1}{2h-2}}) \end{cases} . \quad (13)$$

MANETs: The delay/throughput tradeoff obtained under mobile network is M^2k. It is optimized when $M = \Theta(1)$, with per-node throughput achieves $\Theta(1)$ and the corresponding delay reduced to $\Theta(k)$. Because it can guarantee the maximum number of concurrent active cells as well as the shortest waiting time endured by each node in the cell before transmission or reception.

5.3 Relationship with Other Traffic Patterns and Comparison with Previous Results

When applying Convergimo schemes in both static and mobile networks to them, we can get the throughput and delay of these traffic patterns, as are shown in Table 2. Then, we also make some comparison between our results with those provided in some previous works. The comparison is also shown in Table 2. The throughput in the table is unified to the per-node throughput. For **static** network, our results can achieve the similar performance to unicast presented in [16] and converge-cast ($k = n$) in [17], [18], [19] and [20]. For **MANETs**, a gain of n is achieved on unicast throughput compared with that obtained in [7]. The improvement on throughput is due to our intelligent cooperation between nodes with the help of MIMO. Multiple nodes can transmit simultaneously to other nodes. And a node can successfully decode the original packet once it receives only one combination. The multicast and broadcast result is close to that of [7] with only a $\log n$ factor. Because in such traffic patterns, a source sends identical information to several (or all) destinations. In the scheme of both [7] and ours, although all the destinations can receive the information from their common source within $\log n$ delay, a source has to endure several times' duplication.

6 Conclusion

In this paper, with MIMO, we design two different cooperative schemes for static and mobile ad hoc wireless networks (MANETs), respectively. The hierarchical cooperation scheme under static networks can achieve an aggregate throughput of $\Omega(n^{1-\epsilon})$ for any $\epsilon > 0$. The scheme under MANETs features on joint multiple transmission and reception without hierarchical operations. With optimal network division in the scheme, the achievable per-node throughput can be $\Theta(1)$ with the corresponding delay reduced to $\Theta(k)$.

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Throughput and Delay Analysis for Convergecast with MIMO in Wireless Networks

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1 Some illustrations of Our Convergimo Schemes

1.1 An Illustration of Hierarchical Structure of Convergimo Scheme 1

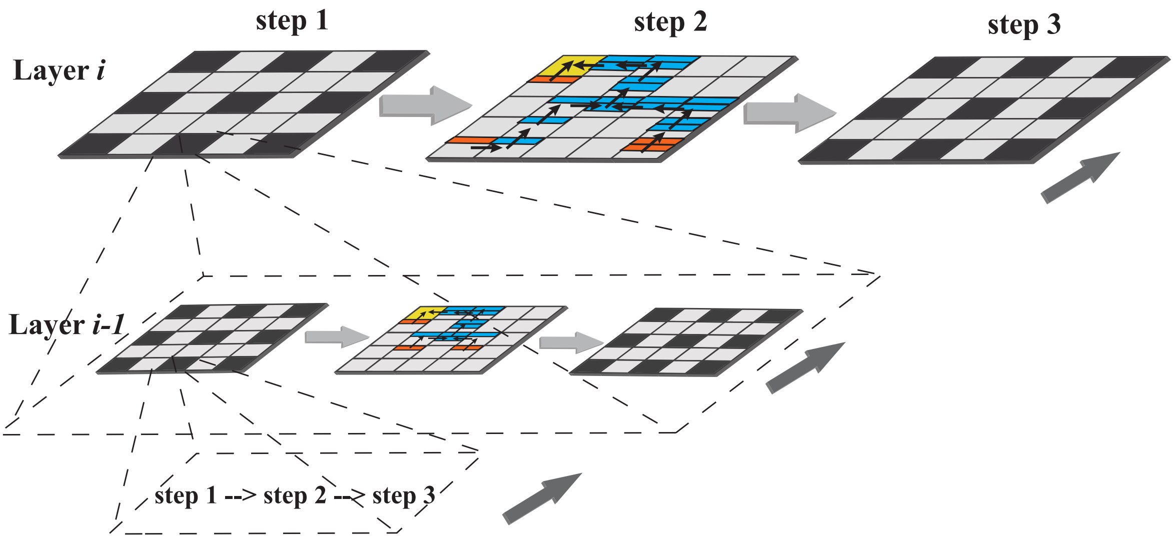


Figure 1: A global view of our hierarchical many-to-one cooperative scheme. The algorithm starts from the bottom layer and keeps executing until it reaches layer h.

1.2 An Illustration of step 2 in Convergimo Scheme 1

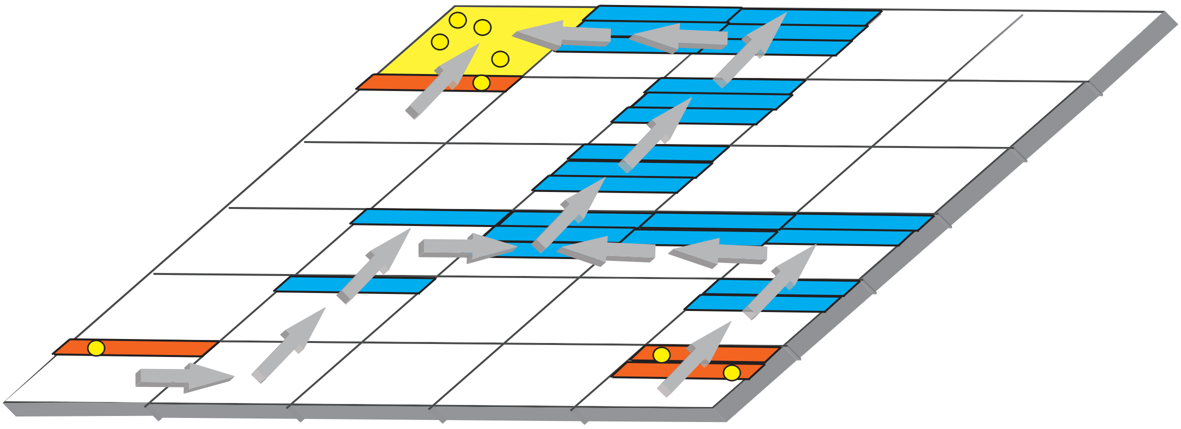
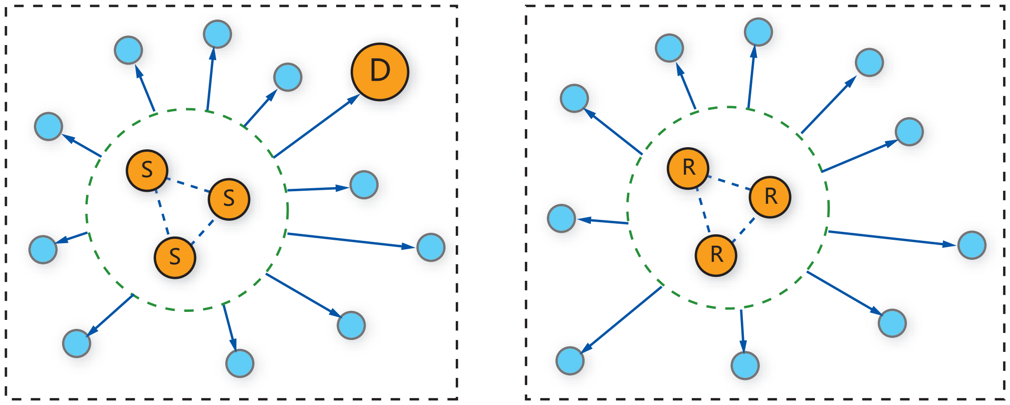


Figure 2: An example of a CT in multi-hop MIMO transmission. Assume one destination has 4 sources. The red parts represent source clusters. 1/4 of the nodes are allowed for transmission in the two source clusters on the left while 1/2 of the nodes are active in the source cluster on the right.Whenever several clusters flow to a common cluster in the next time slot, that cluster will be colored with the part several times' larger than all the transmitting clusters to it. Only the destination cluster (colored with yellow) will be entirely colored when all the data finally flows to it.

1.3 An Illustration of Convergimo Scheme 2



source-destination pair in the cell
(a)

no source-destination pair in the cell
(b)

Figure 3: Illustration of our many-to-one cooperative scheme 2. The unit square is divided into c cells with M nodes located in each cell averagely. The cells colored with yellow can be active concurrently under 9-TDMA scheme. For an active cell, what may be going on is shown in the subfigures (a) and (b) on the right. (a) shows the case where there are at least one sources-destination pairs in the network. (b) shows the case where there are no sources-destination pairs in the cell.

1.4 Proof of Lemma 1

For each destination node in the network, there are k randomly chosen sources belonging to it. If the sets of source nodes for each destination do not intersect with each other, nk nodes will serve as sources in total. However, there are only n nodes in the whole network. Thus, by treating the source-destination pair from a reverse view, for each node s, there are on average k nodes (denoting as $d_1, d_2, \ldots, d_k$) which choose s as one of its source nodes. Assume each source node transmits data to d at data rate λ_i. Since d has to receive k distinctive information from these k sources, it acts as k different nodes during each reception. Thus, the total data rate to the destination d is upper-bounded by the capacity of a multiple-input multiple-output channel between d and the rest of the network. That is,

$$\sum_{i=1}^{k} \lambda_i \le k \log\left(1 + \frac{P}{N_0} \sum_{i=1, s_i \neq d}^{n} \left| \frac{G}{d_{s_i d}^{\alpha}} \right|^2 \right). \tag{1}$$

According to Lemma 1 in [15], the distance between s_i and d is larger than $\frac{1}{n^{1+\delta}}$ *w.h.p.* Thus,

$$\sum_{i=1}^{k} \lambda_i \leq k \log\left(1 + \frac{PG}{N_0} n^{\alpha(1+\delta)+1}\right) \leq Ck \log n, \tag{2}$$

where C is a constant that does not depend on n and k.

If we assume that λ_i, $1 \leq i \leq k$ are identical, then we have

$$\lambda_i \leq C \log n, \tag{3}$$

which implies that

$$\sum_{i=1}^{n} \lambda_i \leq Cn \log n. \tag{4}$$

This completes our proof.

1.5 Proof of Throughput and Delay Provided in Theorem 1

In order to optimize the network division at layer i, we consider two cases, i.e., $n_{c_i} = O(k_i)$ and $n_{c_i} = \Omega(k_i)$. According to Lemma 3 and Lemma 4 in the paper, we have the following two cases:

1. If $n_{c_i} = O(k_i)$, then $k_{c_i} = O(n_{c_i})$, $k_{i-1} = \widetilde{\Theta}\left(\frac{k_i}{n_{c_i}}\right)$;

2. If $n_{c_i} = \Omega(k_i)$, then $k_{c_i} = \Theta(k_i)$, $k_{i-1} = \widetilde{\Theta}(1)$;

In case 1, the throughput in Equation (8) in the paper can be written as

$$\begin{aligned} T_i &= \widetilde{\Theta}\left(\frac{n_i n_{i-1}}{\sqrt{n_i n_{i-1} k_{c_i}} + n_{i-1}^{2-a} k_i^{-b} n_i^b n_{i-1}^{-b}}\right) \\ &= \widetilde{\Theta}\left(\frac{1}{n_{i-1}^{-1} + n_{i-1}^{1-a-b} k_i^{-b} n_i^b}\right). \end{aligned} \tag{5}$$

The result is optimized when T_i is maximized. Taking the derivative of T_i on n_{i-1} and letting $\frac{dT_i}{n_{i-1}} = 0$, w can get $n_{i-1} = \widetilde{\Theta}\left(k_i^{\frac{-b}{a+b-2}} n_i^{\frac{b-1}{a+b-2}}\right)$. Then, $n_{c_i} = \frac{n_i}{n_{i-1}} = k_i^{\frac{b}{a+b-2}} n_i^{\frac{a-1}{a+b-2}} = O(k_i)$ and $k_i = \Omega\left(n_i^{\frac{a-1}{a-2}}\right)$.

At the bottom layer, the aggregate throughput is $\frac{1}{k_1}$. If we divide the network in the optimal way at each layer, the relationship between n_i, k_i and throughput at each layer is $n_i = k_i^{\frac{b-1}{b}} n_{i-1}^{\frac{a+b-2}{b-1}}$ and $T_i = k_i^{\frac{-b}{a+b-2}} n_i^{\frac{b-1}{a+b-2}}$, the recursion calculation is listed as follows:

i	a	b	n_i	T_i
2	0	-1	$k_2^{\frac{1}{2}} n_1^{\frac{3}{2}}$	$k_2^{-\frac{1}{3}} n_2^{\frac{1}{2}}$
3	$\frac{2}{3}$	$-\frac{1}{3}$	$k_3^{\frac{1}{4}} n_2^{\frac{5}{4}}$	$k_3^{-\frac{1}{5}} n_3^{\frac{4}{5}}$
		.		
h	$\frac{2h-4}{2h-3}$	$-\frac{1}{2h-3}$	$k_h^{\frac{1}{2h-2}} n_{h-1}^{\frac{2h-1}{2h-2}}$	$k_h^{-\frac{1}{2h-1}} n_h^{\frac{2h-2}{2h-1}}$

Note that $n_h = n$, we obtain the aggregate throughput at layer h, i.e.,

$$T = \widetilde{\Theta}\left(n^{\frac{2h-2}{2h-1}} \cdot k^{-\frac{1}{2h-1}}\right). \tag{6}$$

In the optimized result, $k = \Omega\left(n^{\frac{a-1}{a-2}}\right) = \Omega\left(n^{\frac{1}{2h-2}}\right)$.

Moreover, since $n_i = k_i^{\frac{1}{2i-2}} n_{i-1}^{\frac{2i-1}{2i-2}}$ and $k_{i-1} = \widetilde{\Theta}\left(\frac{k_i}{n_{c_i}}\right) = \widetilde{\Theta}\left(\frac{k_i}{n_i} \cdot n_{i-1}\right)$, we have $\frac{k_i}{n_i} = \widetilde{\Theta}\left(\frac{k_{i-1}}{n_{i-1}}\right)$ and $\left(\frac{n_i}{k_i}\right) \cdot n_i^{2i-3} = n_{i-1}^{2i-1}$. And this yields to the following derivation:

$$\begin{aligned} n_{i-1} &= \left(\frac{n_i}{k_i}\right)^{\frac{1}{2i-1}} \cdot n_i^{\frac{2i-3}{2i-1}} = \left(\frac{n}{k}\right)^{\frac{1}{2i-1}} \cdot n_i^{\frac{2i-3}{2i-1}} \\ &= \left(\frac{n}{k}\right)^{\frac{1}{2i-1}} \cdot \left(\left(\frac{n}{k}\right)^{\frac{1}{2i+1}} \cdot n_{i+1}^{\frac{2i-1}{2i+1}}\right)^{\frac{2i-3}{2i-1}} \\ &= \left(\frac{n}{k}\right)^{\frac{1}{2i-1}+\frac{1}{2i+1}\frac{2i-3}{2i-1}+\frac{1}{2i+3}\frac{2i-3}{2i+1}\cdots} \cdot n^{\frac{2i-3}{2h-1}}. \end{aligned}$$

Hence,

$$\begin{aligned} n_1 &= \left(\frac{n}{k}\right)^{\frac{1}{3}+\left(\frac{1}{5}\frac{1}{3}+\frac{1}{7}\frac{1}{5}\cdots\frac{1}{2h-1}\frac{1}{2h-3}\right)} n^{\frac{1}{2h-1}} \\ &= \left(\frac{n}{k}\right)^{\frac{1}{3}+\frac{1}{2}\left(\frac{1}{3}-\frac{1}{2h-1}\right)} n^{\frac{1}{2h-1}}, \end{aligned}$$

from which we can obtain $n_1 = n^{\frac{h}{2h-1}} k^{-\frac{h-1}{2h-1}}$.

Now we turn to the analysis on delay at layer i, denoted as $D(i,k)$. Through Equation (**??**), we get $D(h,k) = \frac{nB}{T}$, where B is minimum size of data transmitted at layer h, i.e., $B = \widetilde{\Theta}\left(\prod_{i=1}^{h-1} \frac{n_i}{k_{i+1}}\right) = \widetilde{\Theta}\left(\prod_{i=2}^{h-1} \frac{n_i}{k_i} \cdot \frac{n_1}{k}\right) = \widetilde{\Theta}\left(\left(\frac{n}{k}\right)^{h-2} \frac{n_1}{k}\right) = (n)^{\frac{2h^2-4h+2}{2h-1}} k^{-\frac{2h^2-2h}{2h-1}}$.

And through recursion on $D(i,k)$, the final delay $D(h,k)$ can be obtained, i.e,

$$D(h,k) = \widetilde{\Theta}\left(n^{\frac{2h^2-4h+3}{2h-1}} k^{-\frac{2h^2-2h-1}{2h-1}}\right). \tag{7}$$

Following the same procedure in case 1, the aggregate throughput and delay $D(h,k)$ at layer h can be obtained, which are shown as follows:

$$T = T_h = \widetilde{\Theta}\left(n^{\frac{2h-2}{2h-1}} k^{-\frac{1}{2h-1}}\right), \tag{8}$$

and

$$D(h,k) = \widetilde{\Theta}\left(n^{\frac{h^2-2h+2}{2h-1}} k^{\frac{h^2-4h+3}{2h-1}}\right). \tag{9}$$

1.6 Proof of Lemma 5

Consider a particular cell. Let X_i denote the 0-1 random variable with $X_i = 1$ representing v_i in the cell and $X_i = 0$ representing v_i not in the cell. Note that X_i and X_j are independent with each other. Obviously, $\mathbb{Pr}[X_i = 1] = 1/c$. According to Chernoff bounds, we have

$$\mathbb{Pr}\left\{\sum_{i=1}^{n} X_i > (1+\delta)\frac{n}{c}\right\} \leq e^{-\delta^2 \frac{n}{3c}} \tag{10}$$

and

$$\mathbb{Pr}\left\{\sum_{i=1}^{n} X_i < (1-\delta)\frac{n}{c}\right\} \leq e^{-\delta^2 \frac{n}{2c}}. \tag{11}$$

Hence, with the assumption that $\lim\limits_{n\to\infty} \frac{n}{c\log c} = \infty$, we have

$$\mathbb{Pr}\left\{\left|\sum_{i=1}^{n} X_i - \frac{n}{c}\right| > \delta\frac{n}{c}\right\} \leq e^{-\theta\frac{n}{c}} < e^{-\theta\frac{n}{c\log c}} \to 0 \tag{12}$$

as $n \to \infty$. Thus, $M = \Theta(\frac{n}{c})$ w.h.p. This completes our proof.

1.7 Proof of Lemma 7

Note that for each destination, it has k sources. That is to say, for one session, initially, there are only k nodes which hold the original packets. And the process for letting all the other nodes in the network get these k packets is equivalent to the process for flooding k packets to all the other nodes given that there are originally k distinct nodes holding each of these packets.

First, consider a case where k distinctive packets are stored in k nodes initially and all the other nodes in the network are empty. Now we first analyze the delay required on the process for letting all the nodes in the network have these packets.

Denote J_t as the number of nodes holding the packets from the k nodes at time t. Note that $J_0 = k$. And let $\beta_t = J_t/J_{t-1}$ represent the growth factor after one time slot. Obviously, we have

$$\beta_{t+1} = \frac{J_t + a_1 + a_2 + \ldots + a_{J_t}}{J_t}, \tag{13}$$

where a_i represents the number of new nodes to which the ith packet-holding node transmits during one slot. Note that in each time cell, it is a *multiple-transmission-multiple-reception* process. As the number of packet-holding nodes grows, the growth factor β_t will yield different scale. Also notice that β_t is also influenced by different network division. Jointly consider these two factors, we discuss in the following two cases :

1. If $\frac{n}{M} = \Omega(k)$, then the total number of cells where the k sources are located in, denoted by k_c satisfies $k_c = \Theta(k)$. In this case, initially, J_t is much smaller than $\frac{n}{M}$. Thus, there are on average one such packet-holding node in each cell. This node then transmits the packet to all the other $M-1$ nodes during one time slot. As J_t grows to $\frac{n}{M}$, the number of cells where packet-holding nodes are located becomes the equal order of $\frac{n}{M}$, which can guarantee that there are on average $\Theta\left(\frac{J_t M}{n}\right)$ such packet-holding nodes per cell per time slot.

2. If $\frac{n}{M} = O(k)$, then $k_c = \Theta\left(\frac{n}{M}\right)$: In this case, the initial number of packet holding nodes k is already much larger than the number of cells $\frac{n}{M}$. Thus, the average number of packet-holding nodes per cell is $\Theta\left(\frac{kM}{n}\right)$.

Then, we have the following lemmas:

Lemma *1: *In case 1, for each destination, the time required for all nodes in the network to have one "packet" from the sources corresponding to that destination is*

$$\mathbb{E}[\mathscr{D}_1] = \Theta\left(\frac{\log\left(\frac{n}{k}\right)}{M^{\epsilon}\log(1+M)}\right) + \Theta\left(1 + \frac{1+\log(n-\frac{n}{M})}{M-1}\right), \tag{14}$$

where ϵ is an arbitrarily small value greater than zero.

Proof 1 *Provided in Section 1.8.*

Lemma *2: *In case 2, for each destination, the time required for all nodes in the network to have one "packet" from the sources corresponding to that destination is*

$$\mathbb{E}[\mathscr{D}_1] = \Theta\left(1 + \frac{n\left(1+\log\left(n-k\right)\right)}{M(n-k)}\right). \tag{15}$$

Proof 2 *For case 2, the procedure directly starts from the stage during which the number of nodes grows from k to n. Following the same analysis as that in case 1, we obtain*

$$\mathbb{E}[\mathscr{D}_1] = \Theta\left(1 + \frac{n\left(1+\log\left(n-k\right)\right)}{M(n-k)}\right). \tag{16}$$

Now, we turn back to the real case where all the nodes in the network are destinations, each of which is required to receive packets from its corresponding k sources. Due to the fairness achieved in our scheme, each node in a cell has equal chances to receive packets. Since there are on average M nodes per cell, every node can be active once every M time slots. Therefore, the average delay of the whole network during this period is $\mathbb{E}[D_1] = M \cdot \mathbb{E}\{\mathscr{D}_1\}$. Now we give the following lemma related to $\mathbb{E}[D_1]$.

Lemma *3: *The average delay for letting all nodes in the network to have one "packet" of the sources belonging to the same destination is bounded by*

$$\mathbb{E}[D_1] = \begin{cases} \Theta(M) & \text{if } M = \omega(\log(n)) \\ \Theta(\log n) & \text{if } M = o(\log(n)) \end{cases}. \tag{17}$$

Proof 3 *Provided in Section 1.9.*

1.8 Proof of Lemma *1

For case 1, it is obvious that $\mathbb{E}\{a_i\} \approx M$. Then we have

$$\mathbb{E}\{\beta_{t+1}|J_t\} = \frac{J_t + J_t\mathbb{E}\{a_i\}}{J_t} \approx 1 + M. \tag{18}$$

Since $J_t = \beta_0\beta_1\beta_2 \dots \beta_t$,

$$\mathbb{E}\{J_t\} \approx k(1+M)^t. \tag{19}$$

Note that the period ends when $J_t = \frac{n}{M}$. Denoting D_{1_1} as the time required for k packet-holding nodes to grow to $\frac{n}{M}$, then

$$\mathbb{Pr}[D_{1_1} > t|J_t] \geq \left(1 - \frac{M}{n}\right)^{tJ_t}. \tag{20}$$

Moreover, we have

$$\begin{aligned} \mathbb{E}\{D_{1_1}\} &\geq t\mathbb{Pr}[D_{1_1} > t] \\ &= t\mathbb{E}_{J_t}\{\mathbb{Pr}[D_{1_1} > t|J_t]\} \\ &\geq t\mathbb{E}_{J_t}\left\{\left(1 - \frac{M}{n}\right)^{tJ_t}\right\} \\ &\geq t\left(1 - \frac{M}{n}\right)^{t\mathbb{E}_{J_t}}. \end{aligned} \tag{21}$$

The above inequality holds for all $t > 0$. We choose t to be a base $(1+M)$ logarithm: $t \triangleq \frac{1}{M}\log_{1+M}\left(\alpha(\frac{n}{k})^{M^\delta}\right)$, where α is chosen as $1 \leq \alpha \leq (M+1)$ and

δ is any constant number less than 1. Using this value of t in Inequality (21), we get

$$\begin{aligned}\mathbb{E}\{D_{1_1}\} &\geq \frac{\log(\alpha) + M^{\delta}\log(\frac{n}{k})}{M\log(1+M)}\left[\left(1-\frac{M}{n}\right)^{n}\right]^{\frac{k\left(\alpha(\frac{n}{k})^{M^{\delta}}\right)^{\frac{1}{M}}\log(\frac{n}{k})}{nM\log(1+M)}} \\ &= \frac{\log(\alpha) + M^{\delta}\log(\frac{n}{k})}{M\log(1+M)}\left\{\left[\left(1-\frac{M}{n}\right)^{n}\right]^{\frac{1}{M}}\right\}^{\frac{k\left(\alpha(\frac{n}{k})^{M^{\delta}}\right)^{\frac{1}{M}}\log(\frac{n}{k})}{n\log(1+M)}}.\end{aligned} \tag{22}$$

Note that $\left[\left(1-\frac{M}{n}\right)^{n}\right]^{\frac{1}{M}} \to e^{-1}$ when n goes to infinity. Now we consider the term $\frac{k\left(\alpha(\frac{n}{k})^{M^{\delta}}\right)^{\frac{1}{M}}\log(\frac{n}{k})}{n\log(1+M)}$. Obviously,

$$\frac{k\left(\alpha(\frac{n}{k})^{M^{\delta}}\right)^{\frac{1}{M}}\log(\frac{n}{k})}{n\log(1+M)} < \frac{(n/k)^{M^{-(1-\delta)}}\log(n/k)}{n/k} \to 0$$

as $n \to \infty$. Hence, it follows that

$$\left[\left(1-\frac{M}{n}\right)^{n}\right]^{\frac{k\left(\alpha(\frac{n}{k})^{M^{\delta}}\right)^{\frac{1}{M}}\log(\frac{n}{k})}{nM\log(1+M)}} \to 1.$$

This implies

$$\lim_{n\to\infty}\mathbb{E}\{D_{1_1}\} \geq \frac{\alpha\log\left(\frac{n}{k}\right)}{M^{1-\delta}\log(1+M)}. \tag{23}$$

To obtain a tight bound on Inequality (23), we choose δ to be a value very close to 1. Thus, it can be inferred that

$$\mathbb{E}\{D_{1_1}\} = \Theta\left(\frac{\log\left(\frac{n}{k}\right)}{M^{\epsilon}\log(1+M)}\right), \tag{24}$$

where ϵ is an arbitrarily small value greater than zero.

When J_t grows to $\frac{n}{M}$, unlike the previous process, where $\mathbb{E}[a_i]$ almost remains unchanged per time slot, the duplicate rate $\mathbb{E}[a_i]$ starts to vary in different time slots. Let m denote the number of nodes which do not initially have the packet ($m \leq n - \frac{n}{M}$) and label these m nodes with $\{x_1, x_2, \ldots, x_m\}$. Let X_i represent the number of time slots it takes for the non-packet holding node x_i to reach a cell containing a packet-holding node. Due to the multi-reception, x_i must receive a packet at this time. The probability that at least one of the new node enters the same cell as packet-holding node x_i is $\varphi > 1 - \left(1-\frac{M}{n}\right)^{n-\frac{n}{M}} \geq 1 - e^{1-M}$.

At all times X_i are independent and identically distributed. Denoting D_{1_2} as the time to expand the number of packet-holding nodes from $\frac{n}{M}$ to n, then the random variable D_{1_2} is equal to the maximum value of at most $m = \left\lfloor \frac{n}{M} \right\rfloor$ *i.i.d.* variables. Hence, $\mathbb{E}[D_{1_2}] \leq \mathbb{E}\{\max\{X_1, X_2, \ldots, X_m\}\}$. Now we consider new random variables $\{Y_1, Y_2, \ldots, Y_m\}$ which are assumed to be *i.i.d.* distributed with rate $\nu = \log(1/(1-\varphi))$. Note that $1+Y_i$ is stochastically greater than X_i. Thus, $\mathbb{E}[D_{1_2}] \leq 1 + \mathbb{E}\{\max\{Y_1, Y_2, \ldots, Y_m\}\}$.

Denoting I_i as the interval between the $(i-1)$th and ith completion time. It is easy to know that I_i is the first completion time of $M - i + 1$ racing exponentials. It follows that

$$\mathbb{E}\{I_1 + I_2 + \ldots + I_m\} = \frac{1}{\nu} \sum_{j=1}^{m} \frac{1}{j}. \tag{25}$$

Hence, $\mathbb{E}[D_{1_2}] \leq 1 + \frac{1}{\nu} \sum\limits_{j=1}^{m} \frac{1}{j}$, which is upper bounded by $1 + \frac{1}{\nu}(1 + \log m)$. Therefore,

$$\begin{aligned} \mathbb{E}[D_{1_2}] &\leq 1 + \frac{1}{\nu}(1 + \log m) \\ &\leq 1 + \frac{\left(1 + \log\left(n - \frac{n}{M}\right)\right)}{M-1}. \end{aligned} \tag{26}$$

And

$$\begin{aligned} \mathbb{E}[\mathscr{D}_1] &= \mathbb{E}[D_{1_1}] + \mathbb{E}[D_{1_2}] \\ &= \Theta\left(\frac{\log\left(\frac{n}{k}\right)}{M^{\epsilon}\log(1+M)}\right) + \Theta\left(1 + \frac{1 + \log(n - \frac{n}{M})}{M-1}\right). \end{aligned} \tag{27}$$

1.9 Proof of Lemma *3

For case 1 presented in Lemma 7, we can get

$$M \cdot \mathbb{E}[\mathscr{D}_1] = \Theta\left(\frac{\log\left(\frac{n}{k}\right)}{\log(1+M)}\right) + \Theta\left(M + 1 + \log(n - \frac{n}{M})\right).$$

since $n/M = \Omega(k)$, we have $n/k = \Omega(M)$. Thus, the term $\Theta\left(\frac{\log\left(\frac{n}{k}\right)}{\log(1+M)}\right)$ is less than 1 and is negligible compared to the term $\Theta\left(M + 1 + \log(n - \frac{n}{M})\right)$. Hence, we discuss on the term $\Theta\left(M + 1 + \log(n - \frac{n}{M})\right)$.

- When $M = \Theta(1)$, $\log\left(n - \frac{n}{M}\right)$ approaches to zero. Thus, $M \cdot \mathbb{E}[\mathscr{D}_1] = \Theta(M+1) = \Theta(\log n)$.
- When $M = \omega(1)$, the term $\left(\log\left(n - \frac{n}{M}\right) + 1\right)$ can be omitted. Hence, $M \cdot \mathbb{E}[\mathscr{D}_1] = \Theta(M)$.

For case 2 presented in Lemma 7, we have

$$M \cdot \mathbb{E}[\mathscr{D}_1] = \Theta\left(M + \frac{n(1+\log(n-k))}{n-k}\right).$$

Since $k = o(n)$, $\left(\frac{n(1+\log(n-k))}{n-k}\right) \approx \log(n)$. Therefore, $\mathbb{E}[D_1] = \Theta(M + \log(n))$. With further discussion on M and $\log(n)$, we get

$$M \cdot \mathbb{E}[\mathscr{D}_1] = \begin{cases} \Theta(M) & \text{if } M = \omega(\log(n)) \\ \Theta(\log(n)) & \text{if } M = o(\log(n)) \end{cases}.$$

Jointly consider case 1 and case 2, we have

$$\mathbb{E}[D_1] = \begin{cases} \Theta(M) & \text{if } M = \omega(\log(n)) \\ \Theta(\log(n)) & \text{if } M = o(\log(n)) \end{cases}. \tag{28}$$

This completes our proof of Lemma *3.

1.10 Proof of Lemma 8

We calculate the throughput through the following way: viewing from the perspective of a source node, it belongs to k distinctive destinations on average. Thus, it has to transmit at least k times. Then, the number of transmissions per time slot is $k/(\mathbb{E}(D_N))$. As can be seen from the results on delay, the term Mk dominates the scale of the total delay. And we can simply regard delay as $\Theta(Mk)$. Thus, we obtain the per-node throughput shown in Equation (9).